Mean field approximation for (relatively) small populations

Nicolas Gast

Inria, Grenoble, France (joint work with Benny Van Houdt (Univ. Antwerp))

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Markov models do not scale

- An agent evolves in a finite state-space: $S(t) \in \mathcal{S}$. The system is described by a Kolmogorov equation:

$$\frac{d}{dt}\mathbf{P}\left[S(t) = s\right] = \sum_{s'} \mathbf{P}\left[S(t) = s'\right] Q_{s',s}.$$

Works well if $|\mathcal{S}|$ is "small"

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$\boxed{\begin{array}{l}\text{Problem with population: state space explosion} \\ S \text{ states per agent, } N \text{ agents} \Rightarrow S^N \text{ states}\end{array}}$

Main problem: correlations

$$\mathbf{P}\left[A, B\right] \neq \mathbf{P}\left[A\right]\mathbf{P}\left[B\right]$$

Solution: Mean field approximation, Propagation of Chaos

When a population becomes "large":
(mean field) a single agent has a minor influence on the mass.

$$S_1 \perp\!\!\!\perp \frac{1}{N}\sum_{n=1}^{N}\delta_{S_n} \qquad \text{(as } N \to \infty\text{)}$$

(prop. of chaos) Any finite subset of objects become independent:

$$\mathbf{P}\left[S_1, \dots S_k\right] \approx \mathbf{P}\left[S_1\right] \dots \mathbf{P}\left[S_k\right] \qquad \text{(as } N \to \infty\text{)}$$

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Good. Reduces the complexity from S^N equations to SN!
Bad. Why should this be OK? (or when?)

Discrete space mean field model

Population of N agents

- Each agent evolves in a finite state-space $S_n(t) \in \mathcal{S}$.

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Evolution of *one agent* : Markov kernel $Q(X)$.

$$X_i = \text{fraction of agents in state } i$$

$Q_{ij}(X) =$ rate/proba of one agent of jumping from i to j.

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$Q(.)$ is supposed given and can represent:

- Replicator dynamic, Best-response dynamics
- Effect of environment
- Result of centralized/decentralized optimization

Mean field approximation

When the number of agents is large, agents become independent :

- In the synchronous case[1]:

$$X(t+1) = X(t)Q(X(t))$$

- In the asynchronous case[2].:

$$\frac{d}{dt}X(t) = X(t)Q(X(t))$$

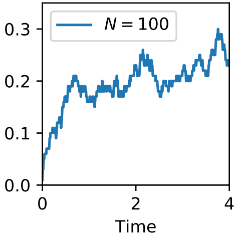
In this talk, I will focus on the latter.

[1] Gomes, Mohr, Souza, 2010 : Discrete time, finite state space mean field games

[2] Gomes,Mohr,Souza 2013: Continuous time finite state mean field game

This talk: relation between finite N models and mean field approximation

$\lim_{N\to\infty}$ (



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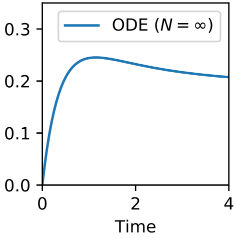
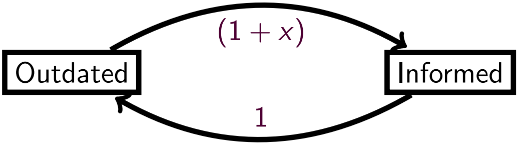


Mean field approximation $\dot{x} = xQ(x)$

$$\mathbf{P}\left[S_n(t) = i\right] \approx X_i(t) \approx x_i(t).$$

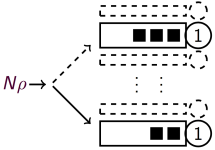
Some examples

Information propagation
x = fraction of "informed" people



Load balancing (super-market model)

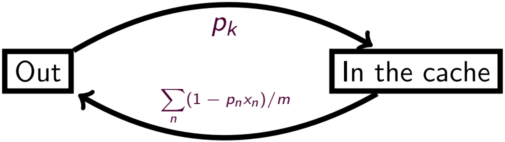
(Mitzenmacher 98, Vvedenskaya 96)



Randomly choose two, and select one

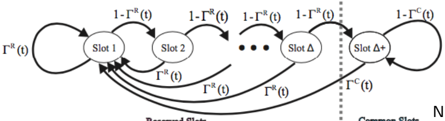
Cache

G.,Van Houdt 2015



802.11 (wireless)

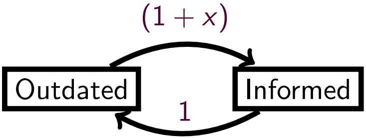
Bianchi 2000, Le Boudec, Cho 2011



Outline

1. Population Processes
2. Moment closure and *refined* mean field approximation
3. Conclusion : Does it always work?

Before studying a generic model, let us look at a simple example



Transitions:

$$X \mapsto X + \frac{1}{N} \quad \text{rate } N(1-X)(1+X)$$

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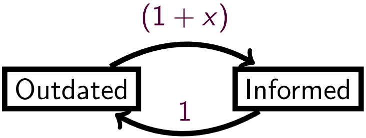

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Drift:

$$\frac{d}{dt}\mathbb{E}\left[X(t)\right] = \mathbb{E}\left[\frac{1}{N}N(1-X)(1+X) - \frac{1}{N}NX\right] = \mathbb{E}\left[1 - X - X^2\right]$$

Mean field approximation:

$$\dot{x} = 1 - x - x^2$$

We study a population of N interchangeable agents

where $O(1)$ agents change states at the same time

X denotes the empirical measure.

$$X_i(t) = \text{ fraction of agents in state } i$$

Transitions are :

$$X \mapsto X + \frac{\ell}{N} \qquad \text{at rate } N\beta_\ell(X).$$

The mean field-approximation is the solution of $\dot{x} = f(x)$ where

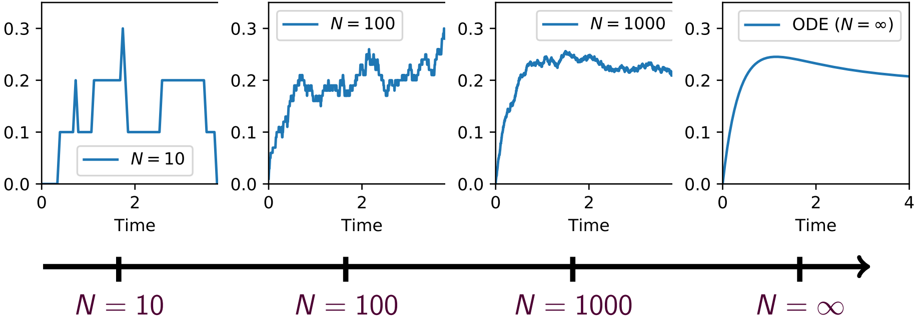
$$f(x) = \sum_{\ell} \ell \beta_\ell(x)$$

[3] $(\mathbf{E}, \|\cdot\|)$ is a subset of a Banach space, typically $\mathbb{R}^d$.

Population processes become deterministic as $N \to \infty$

Theorem (Kurtz (1970s), Ying (2016)):

If the drift f is Lipschitz-continuous:	If in addition the ODE has a unique attractor π:
$X^N(t) \approx x(t) + \frac{1}{\sqrt{N}} G_t$	$\mathbb{E}\left[X^N(\infty) - \pi\right] = O(1/\sqrt{N})$



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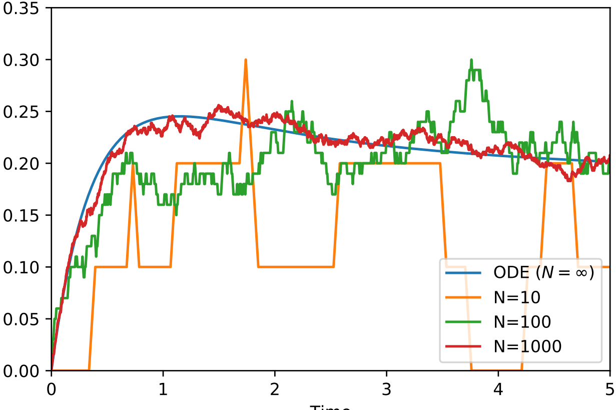


Take-home message

For a population process with homogeneous interactions:

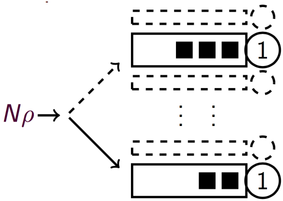
- The mean field approximation is asymptotically exact
  - Functional law of large number
- The population X is at distance $1/\sqrt{N}$ from the mean field.
  - Functional central limit theorem

Outline

What changes when one focus on performance evaluation?

Simulations results ($\rho = 0.9$)

N	10	100	1000	∞ (mean field)
Average queue length (simu.)	2.8040	2.3931	2.3567	2.3527
Error of mean field	0.4513	0.0404	0.0040	0



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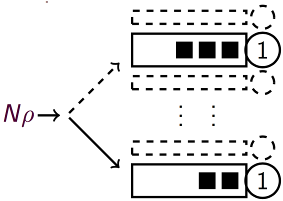

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Theorem (Kolokoltsov 2012, G. 2017& 2018). If the drift f is C^2 and has a unique exponentially stable attractor, then for any $t \in [0, \infty) \cup \{\infty\}$, there exists a constant V_t such that:

$$\mathbb{E}\left[h(X^N(t))\right] = h(x(t)) + \frac{V(t)}{N} + O(1/N^2)$$

Where does the $1/N$-term comes from?

The moment closure approach

Going back to the information propagation example (and writing X instead of $X(t)$, we get:

$$\frac{d}{dt}\mathbb{E}[X] = \mathbb{E}\left[1 - X - X^2\right]$$
$$= 1 - \mathbb{E}[X] - \mathbb{E}\left[X^2\right]$$

Problem: this equation is not closed because we need $\mathbb{E}\left[X^2\right]$.

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Hence, there are two choices:

1. Assume $\mathbb{E}\left[X^2\right] \approx \mathbb{E}\left[X\right]^2$. This gives the mean field approximation:

$$\dot{x} = 1 - x - x^2.$$

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2. Obtain an equation for $\mathbb{E}\left[X^2\right]$.

$$X^2 \mapsto (X + \frac{1}{N})^2 \text{ at rate } N(1 - X^2)$$
$$X^2 \mapsto (X - \frac{1}{N})^2 \text{ at rate } X$$

The moment closure approach (continued)

Hence on average:

$$\begin{aligned}
\frac{d}{dt}\mathbb{E}\left[X^2\right] &= \mathbb{E}\left[(\frac{2X}{N}+\frac{1}{N^2})N(1-X^2)+(-\frac{2X}{N}+\frac{1}{N^2})NX)\right] \\
&= \mathbb{E}\left[2X-2X^3-2X^2+\frac{1}{N}(1-X^2+X)\right] \\
&= 2\mathbb{E}\left[X\right]-2\mathbb{E}\left[X^3\right]-2\mathbb{E}\left[X^2\right]+\frac{1}{N}(1-\mathbb{E}\left[X^2\right]+\mathbb{E}\left[X\right])
\end{aligned}$$

Problem: this equation is not closed because we need $\mathbb{E}\left[X^3\right]$.

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$$= \mathbb{E}\left[2X-2X^3-2X^2+\frac{1}{N}(1-X^2+X)\right]$$

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Problem: this equation is not closed because we need $\mathbb{E}\left[X^3\right]$.

Hence, there are two choices:

1. Assume $\mathbb{E}\left[X^3\right] \approx 3\mathbb{E}\left[X^2\right]\mathbb{E}\left[X\right]-2\mathbb{E}\left[X\right]^3$. This gives the second order moment closure approximation:

$$\dot{x} = 1-x-y$$

$$\dot{y} = 2x-(3xy-2x^3)-2y+\frac{1}{N}(1-y+x)$$

2. Obtain an equation for $\mathbb{E}\left[X^3\right]$ (that will involve $\mathbb{E}\left[X^4\right]$...)

Using this approach, we can derive $1/N^k$-expansions

Theorem. Assume that f is C^2 and let x be the solution of $\frac{d}{dt}x = f(x)$.

$$\frac{d}{dt}\mathbb{E}\left[X(t)\right] = x(t) + O(1/N).$$

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Let $Y(t) = X(t) - x(t)$. Then :

$$\mathbb{E}\left[Y(t)\right] = \frac{1}{N}V(t) + O(1/N^2)$$

$$\mathbb{E}\left[Y(t)\otimes Y(t)\right] = \frac{1}{N}W(t) + O(1/N^2)$$

where

$$\frac{d}{dt}V^i = f^i_j V^j + f^i_{j,k}W^{j,k}$$

$$\frac{d}{dt}W^{j,k} = f^j_\ell W^{\ell,k} + f^k_\ell W^{j,\ell}$$

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$$\mathbb{E}\left[Y(t) \otimes Y(t)\right] = \frac{1}{N}W(t) + \frac{1}{N^2}B(t) + O(1/N^3)$$

$$esp Y(t)^{\otimes 3} = \frac{1}{N^2}C(t) + O(1/N^3)$$

$$esp Y(t)^{\otimes 4} = \frac{1}{N^2}D(t) + O(1/N^3)$$

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$$\frac{d}{dt}W^{j,k} = f^j_\ell W^{\ell,k} + f^k_\ell W^{j,\ell}$$

$$\frac{d}{dt}A^i = f^i_j A^j + f^i_{j,k}B^{j,k} + f^i_{j,k,\ell}C^{j,k,\ell} + f^i_{j,k,\ell,m}D^{j,k,\ell,m}$$

$$\frac{d}{dt}B^{i,j} = f^i_k B^{k,j} + f^j_k B^{k,j} + \frac{3}{2}\left[f^i_{k,\ell}C^{k,\ell,j} + f^j_{k,\ell}C^{k,\ell,i}\right] + 2(f^i_{k,\ell,m}D^{k,\ell,m,j} + f^j_{k,\ell,m}D^{k,\ell,m,i}) + \frac{1}{2}Q^{i,j}_k V^k + \frac{1}{2}Q^{i,j}_{k,\ell}W^{k,\ell}$$

...

Computational issues

Recall that $x(t)$ be the mean field approximation and $Y(t) = X(t) - x(t)$.

You can close the equations by assuming that $Y^{(k)} = 0$ for $k > K$.

- For $K = 0$, this gives the mean field approximation ($1/N$-accurate)
- For $K = 2$, this gives the refined mean field ($1/N^2$-accurate).
- For $K = 4$, this gives a second order expansion ($1/N^3$-accurate).

For a system of dimension d, $Y(t)^{(k)}$ has d^k equations.

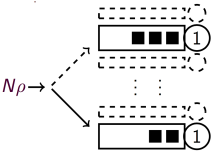
Computational issues

- The mean field is a system of non-linear ODE of dimension d (where $d = |\mathcal{S}|$ if all agents have the same parameters and $N|\mathcal{S}|$ if they are all different)
- The $1/N$ term adds two systems of **time-inhomogeneous linear** ODEs of dimension d^2 and d.
- The $1/N^2$ term adds four systems of **time-inhomogeneous linear** ODEs of dimension d^4, d^3, d^2 and d.

To compute, you essentially need up to the second (for the $1/N$-term) or the fourth (for the $1/N^2$-term) derivatives of the drifts.

We implemented this is a numerical library

https://github.com/ngast/rmf_tool/



Randomly choose two, and select one

The transitions are (for $i \in \{1 \dots K\}$):

$$+\frac{1}{N}\mathbf{e}_i \text{ rate } N\rho(x_{i-1}^2 - x_i^2)$$

$$-\frac{1}{N}\mathbf{e}_i \text{ rate } N(x_i - x_{i+1})$$

```
ddpp = rmf.DDPP()
K = 20 # we truncate at 20

# The vector 'e(i)' is a vector where only the $i$th coordinate equals $1$
def e(i):
    l = np.zeros(K)
    l[i] = 1
    return(l)

# We then add the transitions :
for i in range(K):
    if i>=1:
        ddpp.add_transition(e(i),
            eval('lambda x: rho*(x[{}]*x[{}] - x[{}]*x[{}] )'.format(i-1,i-1,i,i) ))
    if i<K-1:
        ddpp.add_transition(-e(i),
            eval('lambda x: (x[{}] - x[{}])'.format(i,i+1) ))
ddpp.add_transition(e(0), lambda x : rho*(1-x[0]*x[0]))
ddpp.add_transition(-e(K-1), lambda x : x[K-1])
```

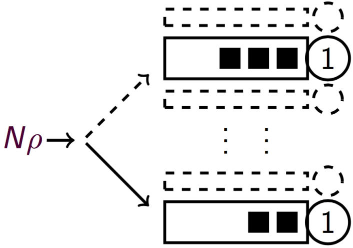
```
ddpp.set_initial_state(e(0)) # initial state

print('\t\t      N=10\t     N=50\t    N=inf',end='')
for rho in [0.7,0.9,0.95]:
    print('\nrho=',rho,'\t',end=' ')
    x = ddpp.fixed_point()
    c = ddpp.theoretical_C()
    for N in 10,50,np.inf:
        print(sum(x+c/N),end=' ')
```

```
                 N=10          N=50          N=inf
rho= 0.7         1.21502419299 1.14709894998 1.13011763922
rho= 0.9         2.75129433831 2.43238017933 2.35265163959
rho= 0.95        4.10172926564 3.39146081504 3.21389370239
```

The refined mean field approximation is very accurate

... when predicting steady-state performance



Randomly choose two, and select one

Arrival at each server ρ.

- Sample $d-1$ other queues.
- Allocate to the shortest queue

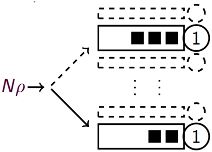
Service rate$=1$.

	$N=10$	$N=20$	$N=50$	$N=100$
Mean Field	2.3527	2.3527	2.3527	2.3527
$1/N$-expansion	2.7513	2.5520	2.4324	2.3925
$1/N^2$-expansion	2.8045	2.5653	2.4345	2.3930
Simulation	2.8003	2.5662	2.4350	2.3931

Steady-state average queue length ($\rho=0.9$).

The refined mean field approximation is very accurate

... to evaluate the transient performance



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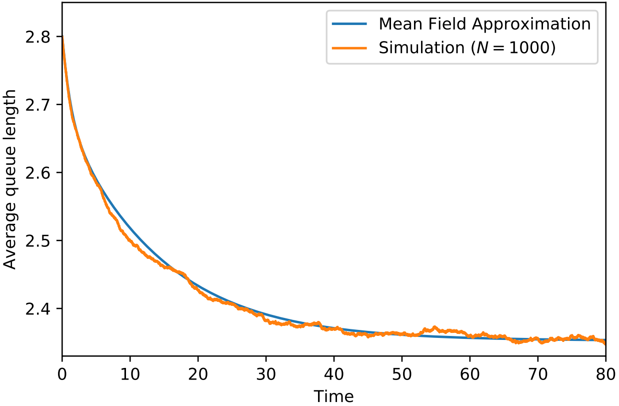

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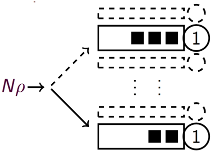

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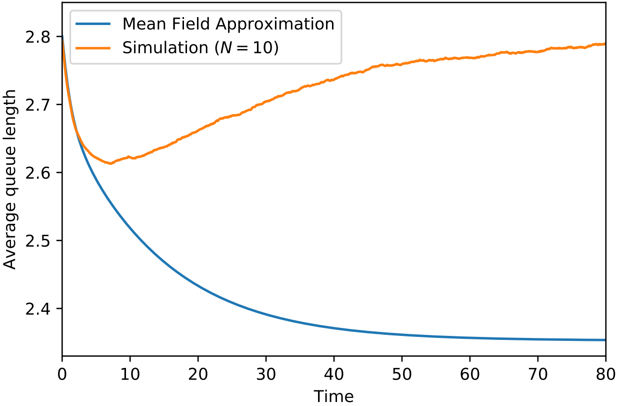

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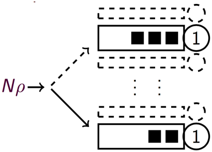

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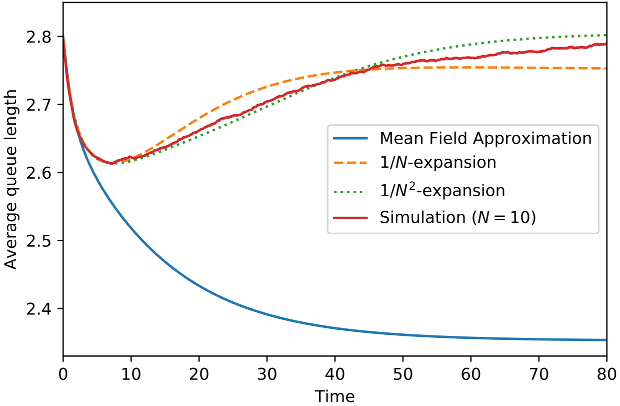


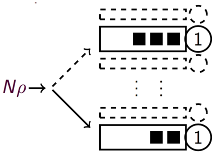
Remark about computation time :

- 10min/1h (simulation $N = 1000/N = 10$), C++ code. Requires many simulations, confidence intervals,...
- 80ms (mean field), 700ms ($1/N$-expansion), 9s ($1/N^2$-expansion), Python `numpy`

The refined approximation can also account for behaviors that are indistinguishable by classical mean field methods

Example: choosing with or without replacement

Let x_i be the fraction of servers with i or more jobs. Pick two servers, what is the probability that the least loaded has exactly i jobs?



Randomly choose two, and select one

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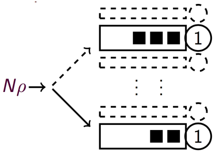

Randomly choose two, and select one

- If picked with replacement: $x_i^2 - x_{i+1}^2$.
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The two coincide as $N \to \infty$.

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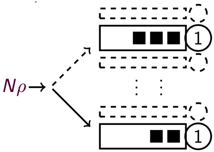

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$N = 10$ servers		Simulation	Refined mean field	Mean field
$\rho = 0.9$	with	2.820	2.751	2.3527
	without	2.705	2.630	2.3527
	with-without	0.115	0.121	–

Outline

Recap and extensions

If you fix a control policy such that $x \mapsto xQ(x)$ is C^2, then :

1. The accuracy of the classical mean field approximation is $O(1/N)$.
   - Mean field approximation = propagation of chaos (= independence)
2. We can use this to define a refined approximation.
   - Refined mean field approximation = look at covariance
3. The refined approximation is often accurate for $N = 10$:

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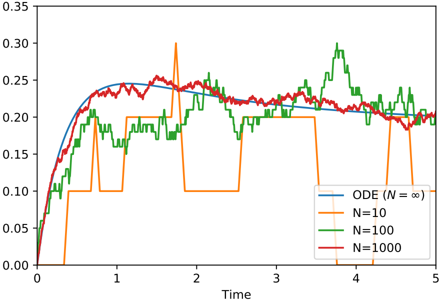
Extensions:

- Transient regime
- Discrete-time systems
- We can also compute the next term in $1/N^2$.

Limit 1: it applies to agent properties but not to populations

Population's state: $X(t) = \frac{1}{N}\sum_{n=1}^{N} \delta_{S_n(t)}$

$$X(t) = x(t) + \frac{G(t)}{\sqrt{N}}$$



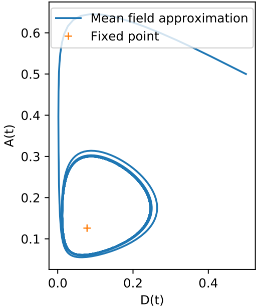
One agent has state $S_n(t)$

$$\mathbb{E}\left[X(t)\right] = x(t) + \frac{C}{N}$$

Average queue length
($N = 10$ and $\rho = 0.9$)

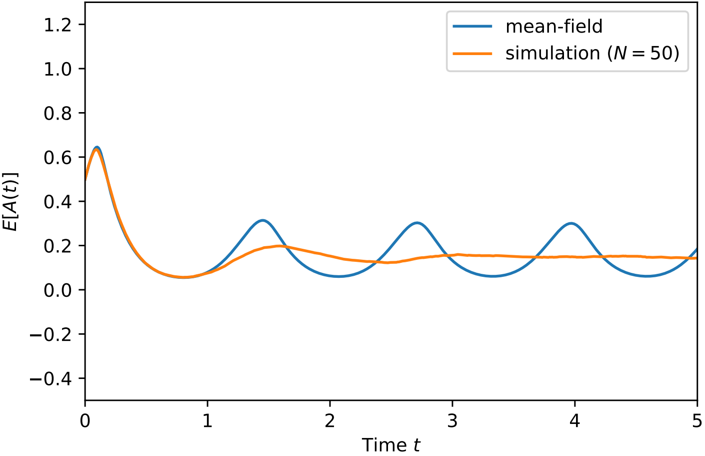
Simu	Refined M.F.	M.F.
2.804	2.751	2.353

Limit 2: It can fail when the mean field approximation has limiting cycles



Transition	Rate
$(D,A,S) \mapsto (D-\frac{1}{N}, A+\frac{1}{N}, S)$	$N(0.1+10X_A)X_D$
$(D,A,S) \mapsto (D, A-\frac{1}{N}, S+\frac{1}{N})$	$N5X_A$
$(D,A,S) \mapsto (D+\frac{1}{N}, A, S-\frac{1}{N})$	$N(1+\frac{10X_A}{X_D+\delta})X_S$

Limit 2: It can fail when the mean field approximation has limiting cycles



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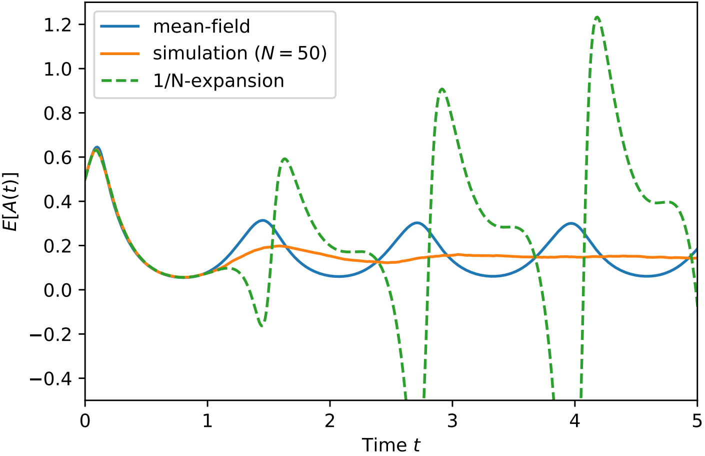

Limit 3: What about games and/or optimal control?

Discrete-state mean field games are relatively “easy” to work with.

- Forward equation : ODE.
- Backward equation : MDP (Markov decision process)

Open question : Do the Nash equilibria of the finite games converge to a mean field equilibria? What is the rate of convergence?

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- There are examples with refined-$1/N$ equilibrium (see Gueant et al. "when does the meeting start")
- The value of the game does not always converge (Doncel et al. 2017)
- When it does, convergence is often at rate $O(1/\sqrt{N})$.

Some References

http://mescal.imag.fr/membres/nicolas.gast

nicolas.gast@inria.fr

- A Refined Mean Field Approximation by Gast and Van Houdt. SIGMETRICS 2018 (best paper award)
- Size Expansions of Mean Field Approximation: Transient and Steady-State Analysis Gast, Bortolussi, Tribastone
- Expected Values Estimated via Mean Field Approximation are O(1/N)-accurate by Gast. SIGMETRICS 2017.
- https://github.com/ngast/rmf_tool/