If agents could talk…. what should they say?

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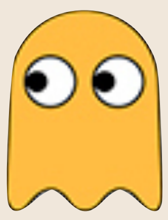
University of California, Santa Barbara

(joint with D. Grimsman, M.S. Ali, D. Paccagnan, V. Ramaswami, and J. Hespanha)

Workshop on Network, Population and Congestion Games
Institut of Henri Poincaré, Paris
April 17, 2019



Central Goal

design of ***admissible*** control algorithms that attain ***near-optimal*** system-wide behavior in a ***reasonable*** period of time

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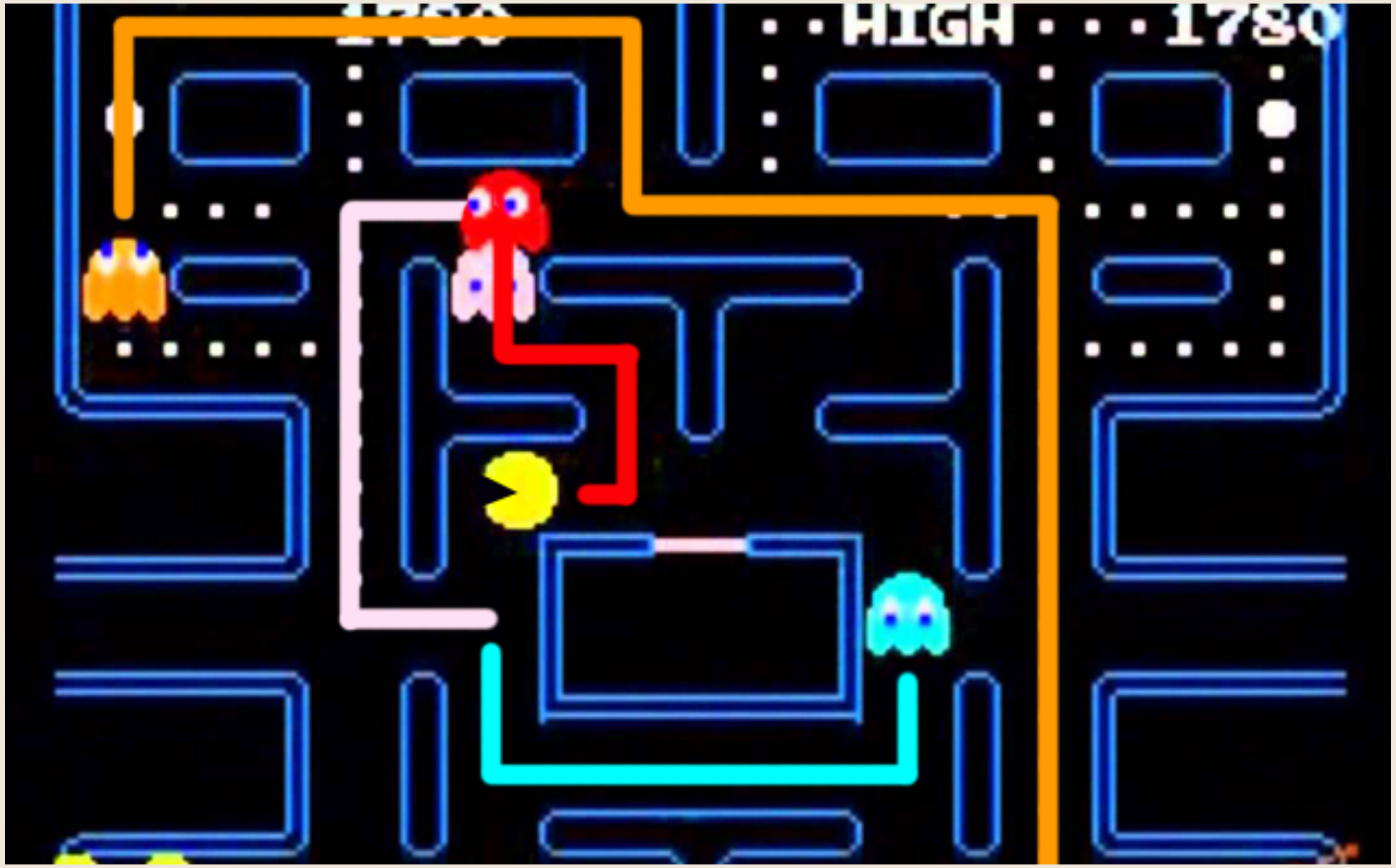

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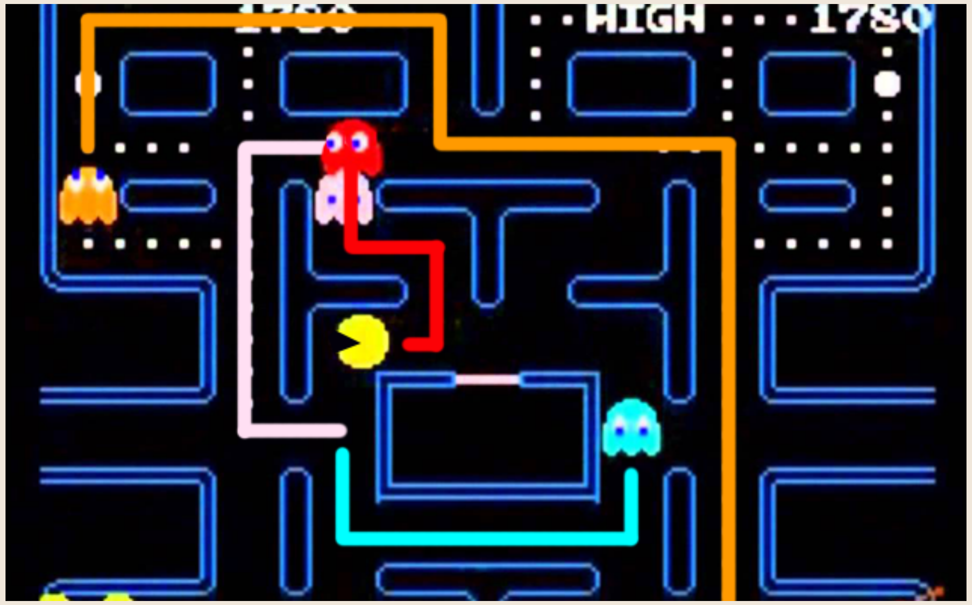
design of ***admissible*** control algorithms that attain ***near-optimal*** system-wide behavior in a ***reasonable*** period of time

admissible: centralized/decentralized, informational dependence, etc.



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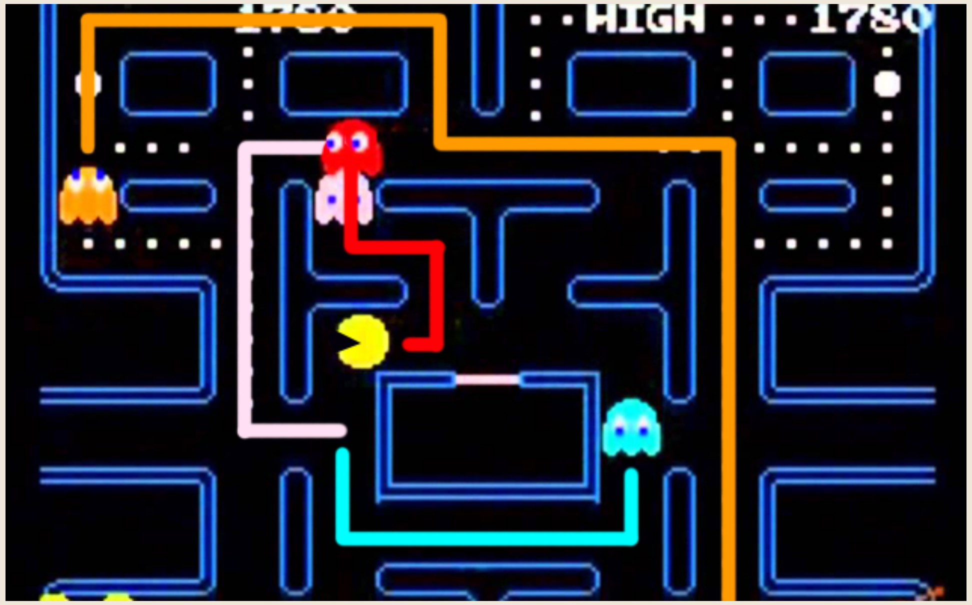
near-optimal: attainable performance close to best centralized algorithm

$$1 \geq \frac{W(\text{control algorithm})}{W(\text{best centralized})} \geq 0$$



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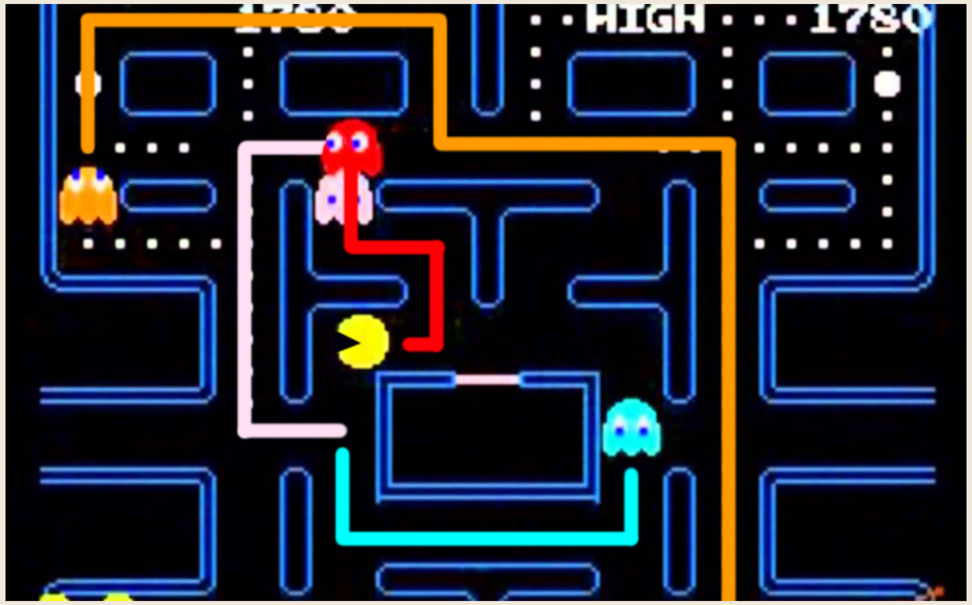
reasonable: linear or polynomial in the system dimensions



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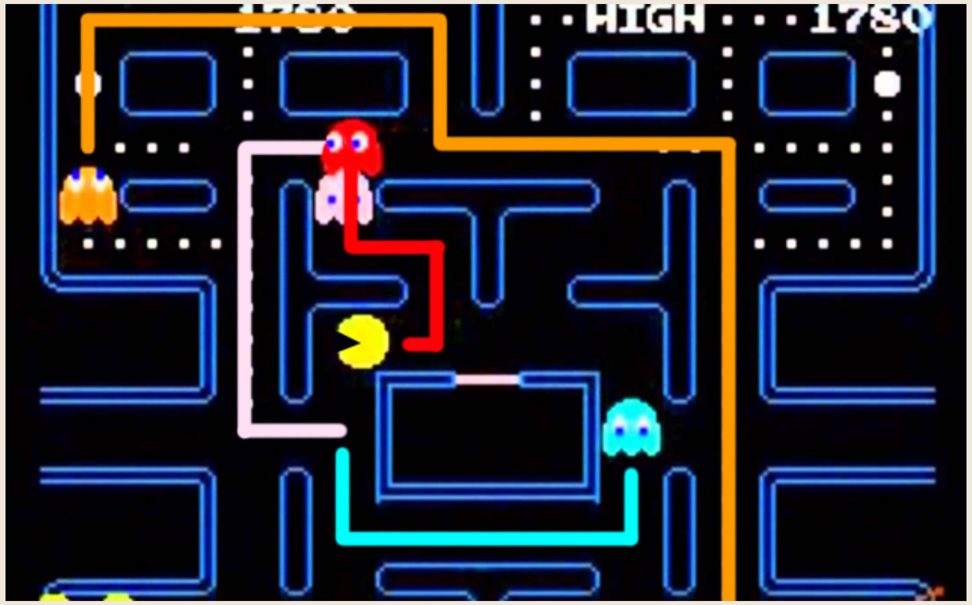
relationship well studied
(computer science / optimization)



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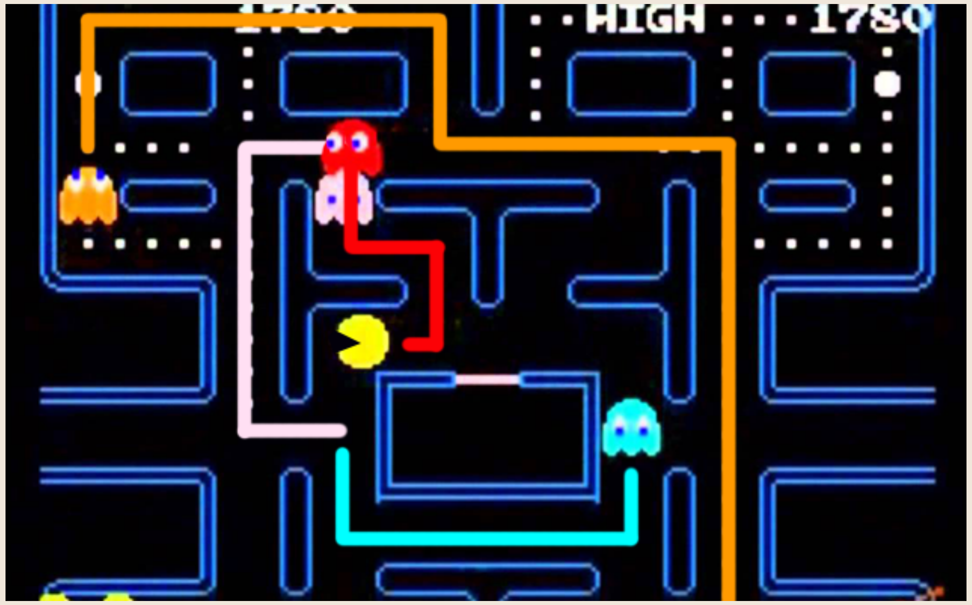
how does *informational availability* impact achievable performance guarantees?



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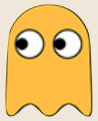
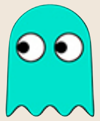
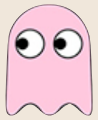
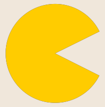
What information does  have?

- , ,  planned paths?
-  location?
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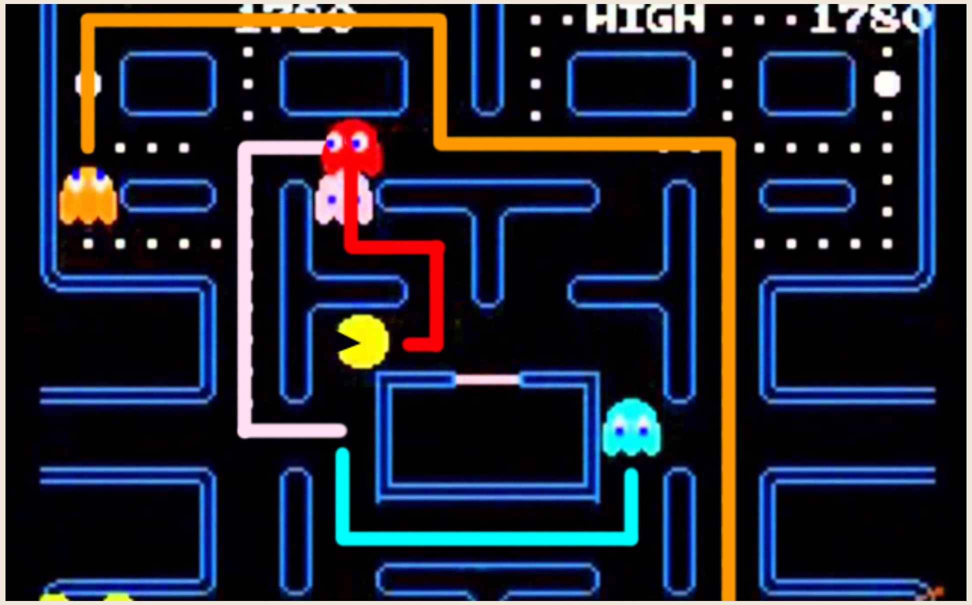

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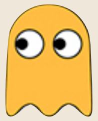
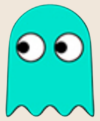
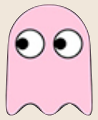
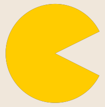
What information does  have?

- , ,  planned paths?
-  location?
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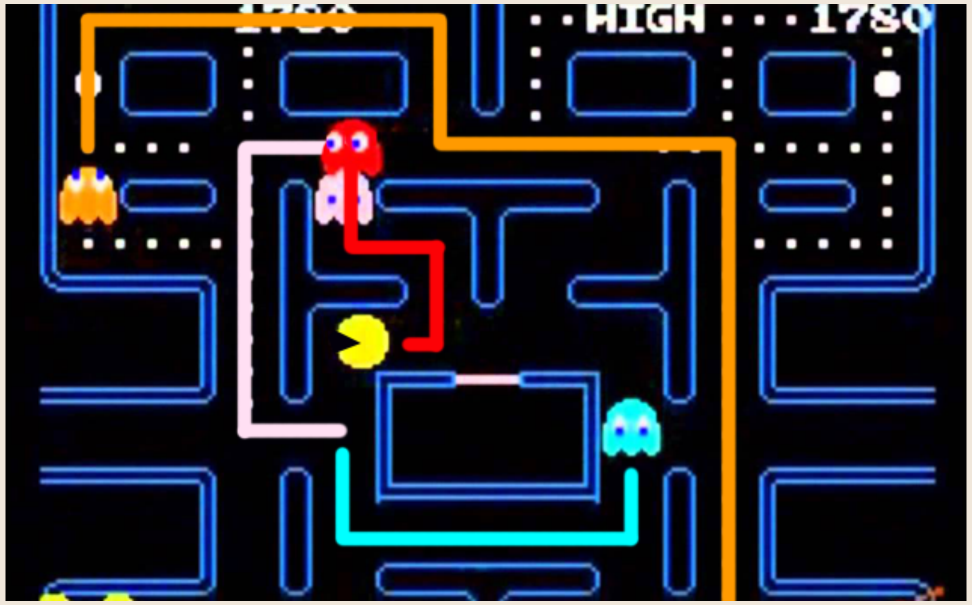
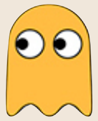
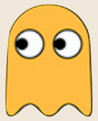


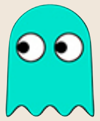
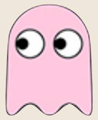
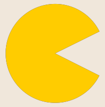
What should  do with this information?

Design Challenges

Transcription

Policy generation

What information does  have?

- , ,  planned paths?
-  location?
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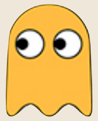
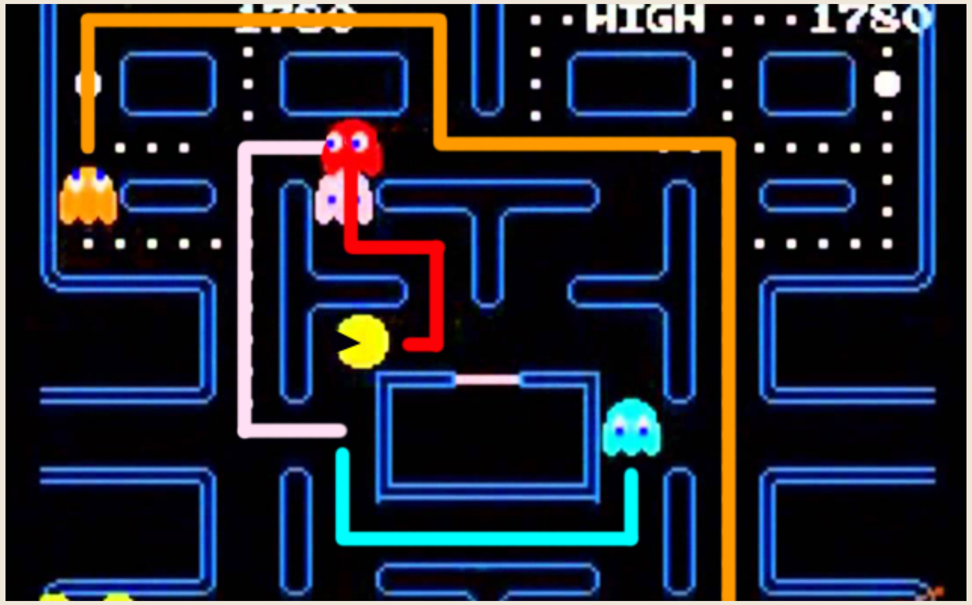
What should  do with this information?



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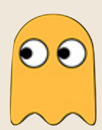
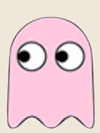
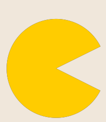
What information does  have?

- , ,  planned paths?
-  location?
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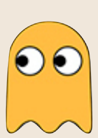
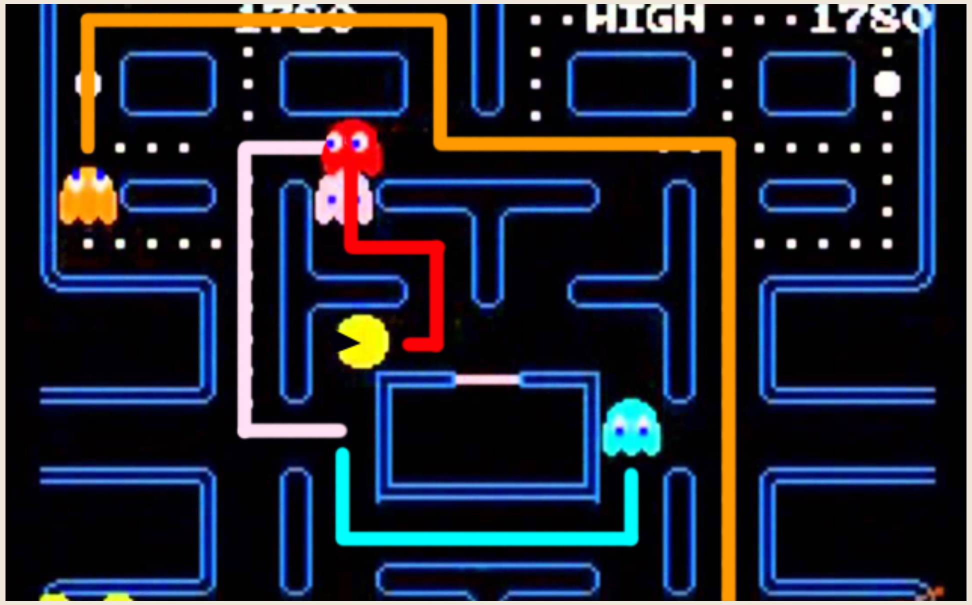
What should  do with this information?



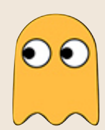
Design Challenges

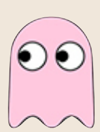
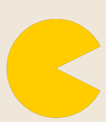
Transcription

Policy generation

Part I

How does the lack of information degrade achievable performance?

What information does  have?

- , ,  planned paths?
-  location?
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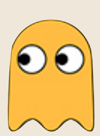
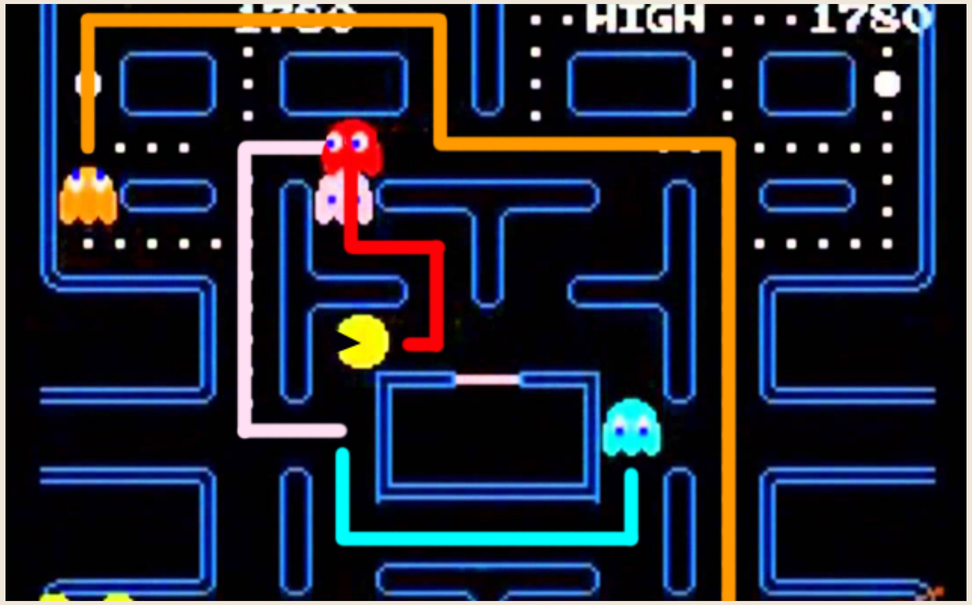
What should  do with this information?



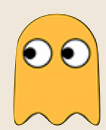
Design Challenges

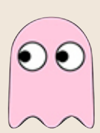
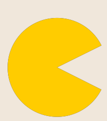
Transcription

Policy generation

Part I
How does the lack of information degrade achievable performance?

Part II
How do you optimize collective performance using available information?

What information does  have?

- , ,  planned paths?
-  location?
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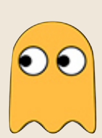
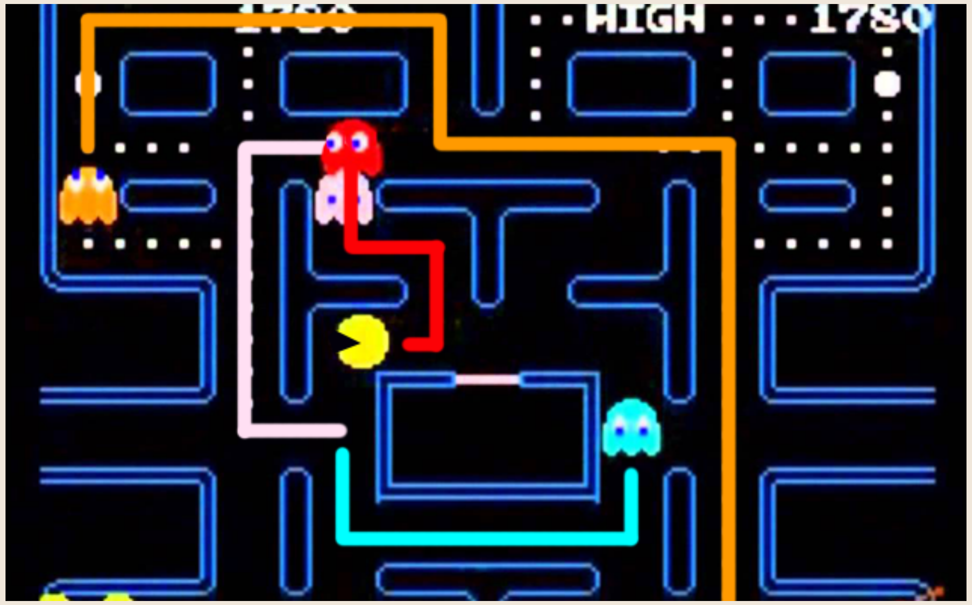
What should  do with this information?



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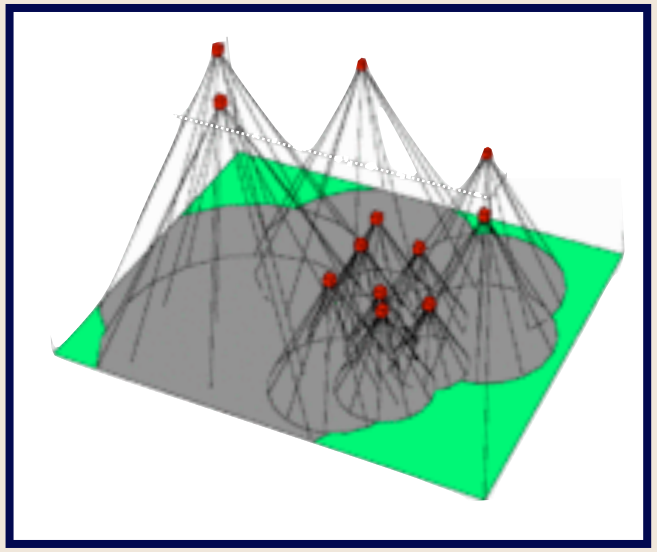
submodularity

(diminishing returns)

Sensor coverage



(many engineering problems are submodular)

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submodularity **+** ***greedy algorithm***

$$\frac{W(\text{greedy algorithm})}{W(\text{best centralized})} \geq \frac{1}{2}$$

[Fisher et al., 1978]

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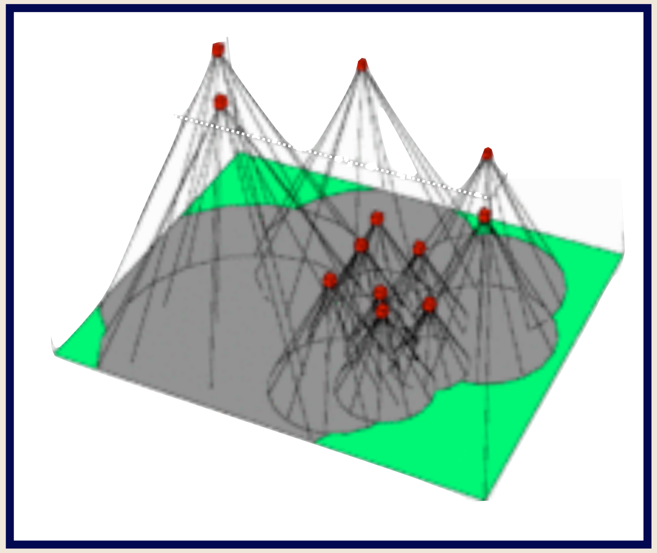
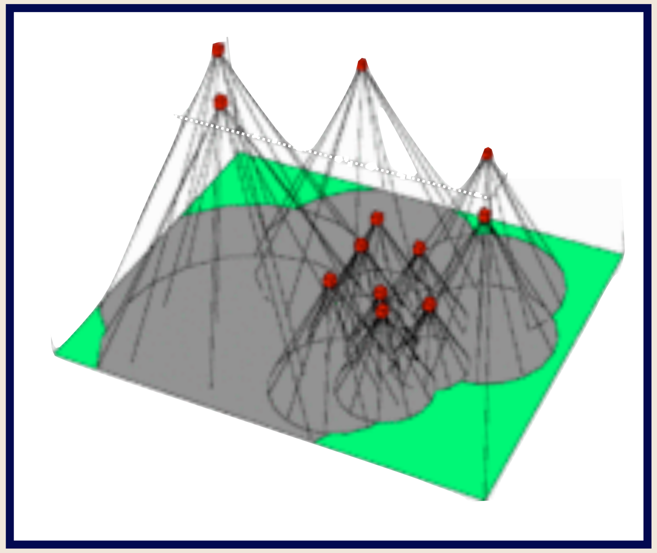


- Elements: E
- Welfare function: $W : 2^E \to \mathbb{R}$

Setup:



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Properties:

- Normalized: $W(\emptyset) = 0$

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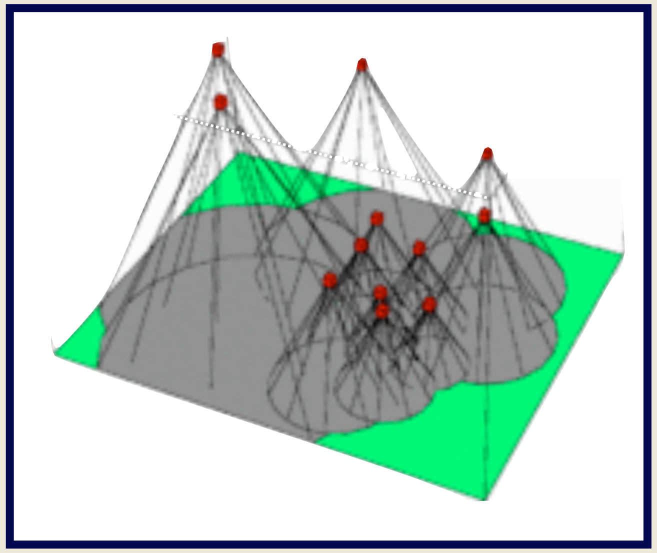

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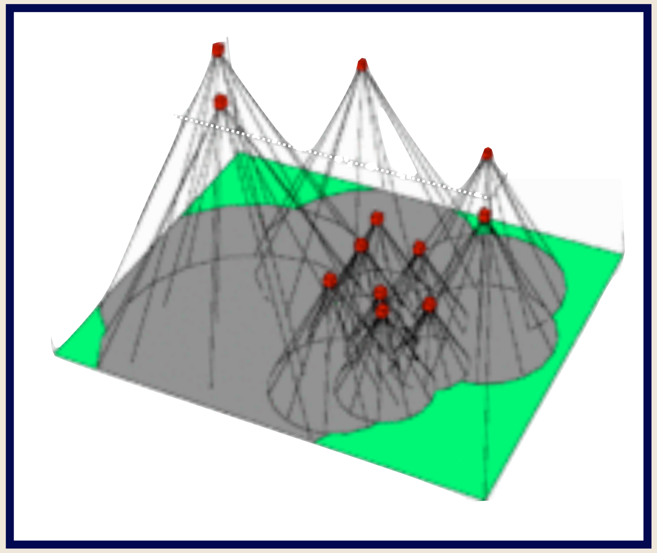


Properties:

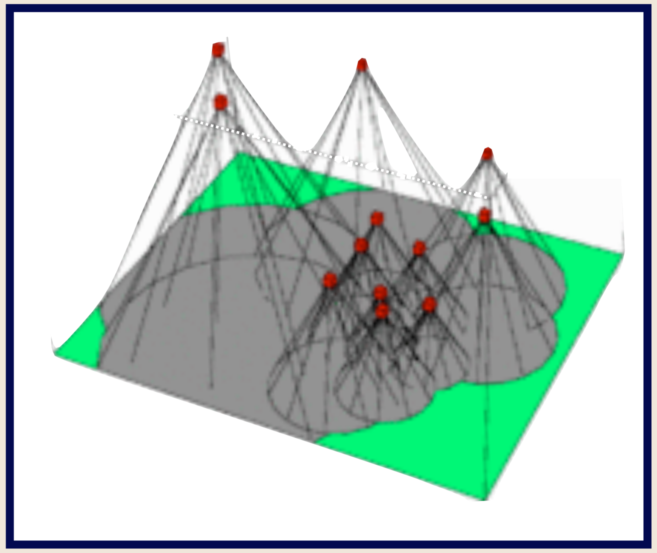
- Normalized: $W(\emptyset) = 0$
- Monotone: $W(A) \leq W(B), \ \forall A \subseteq B \subseteq E$
- Submodular: For any $A \subseteq B \subseteq E, \ x \in E \setminus B$

$$W(A \cup \{x\}) - W(A) \geq W(B \cup \{x\}) - W(B)$$

marginal gain adding x to "smaller" set A

marginal gain adding x to "larger" set B

Setup:



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marginal gain adding x to "larger" set B

Note: We will refer to such as function as merely submodular

Setup:

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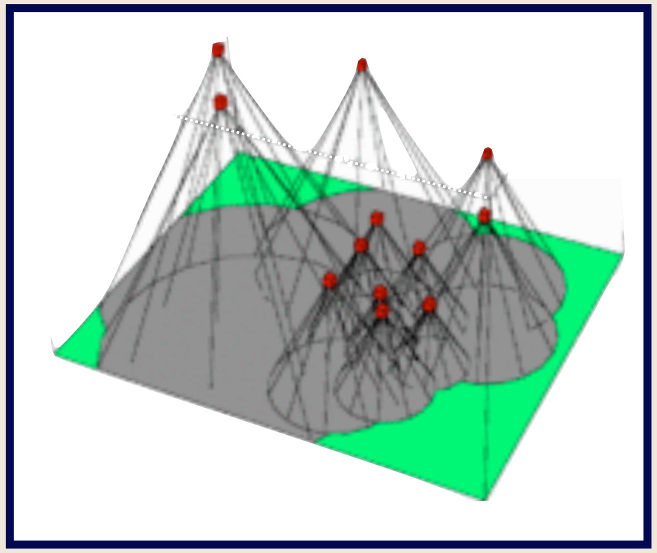

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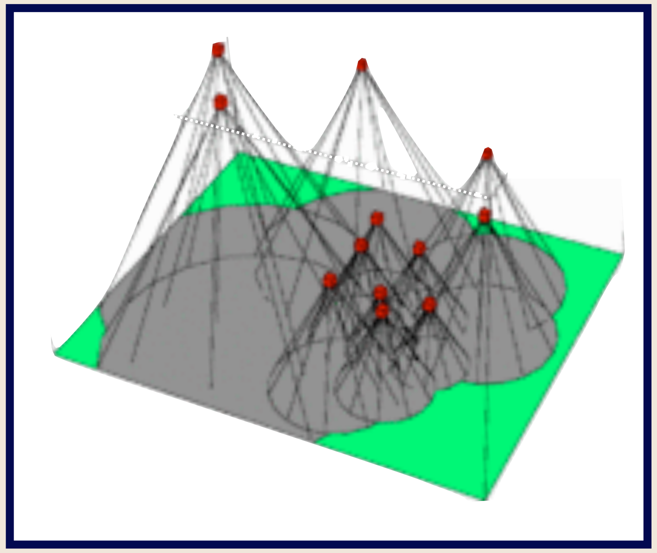

Multiagent Setup:

- Agents: $N = \{1, 2, \ldots, n\}$

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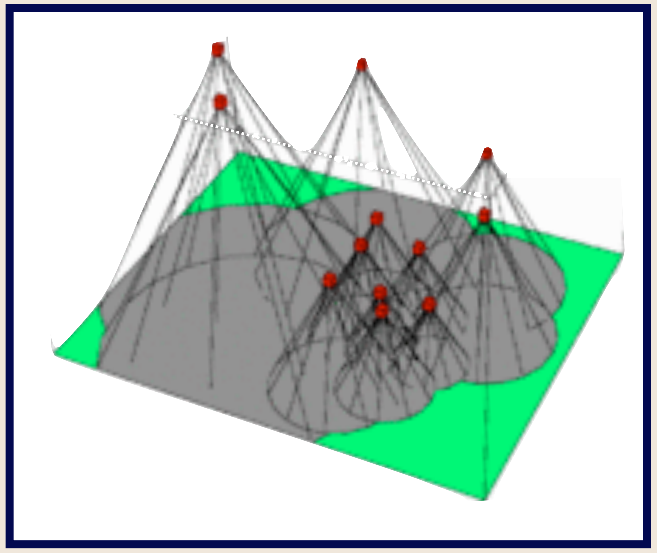
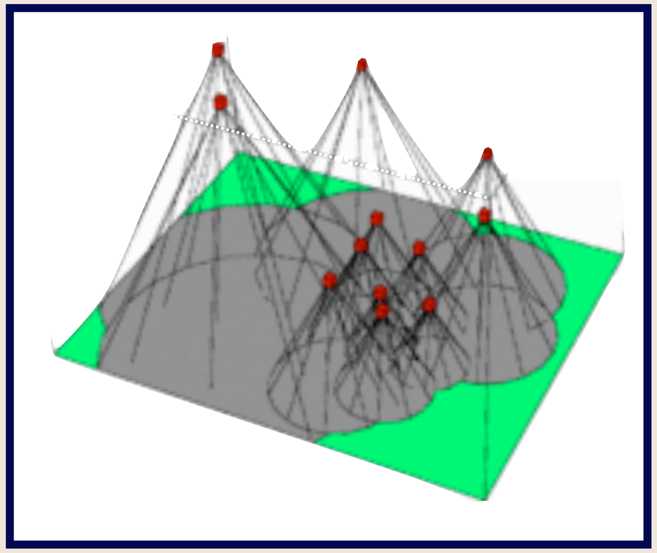


Multiagent Setup:

- Agents: $N = \{1, 2, \ldots, n\}$
- Choices: $x_i \in X_i \subseteq 2^E$

Setup:



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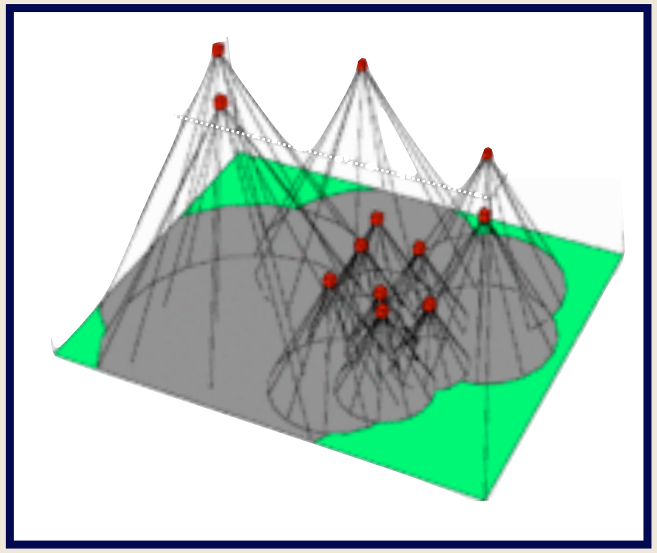

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Example: Coverage

- Agents: *Sensors*
- Choices: *Local coverage area*
- Evaluation: *Joint coverage quality*

submodularity + **greedy algorithm**

$$\frac{W(\text{greedy algorithm})}{W(\text{best centralized})} \geq \frac{1}{2}$$

Setup:

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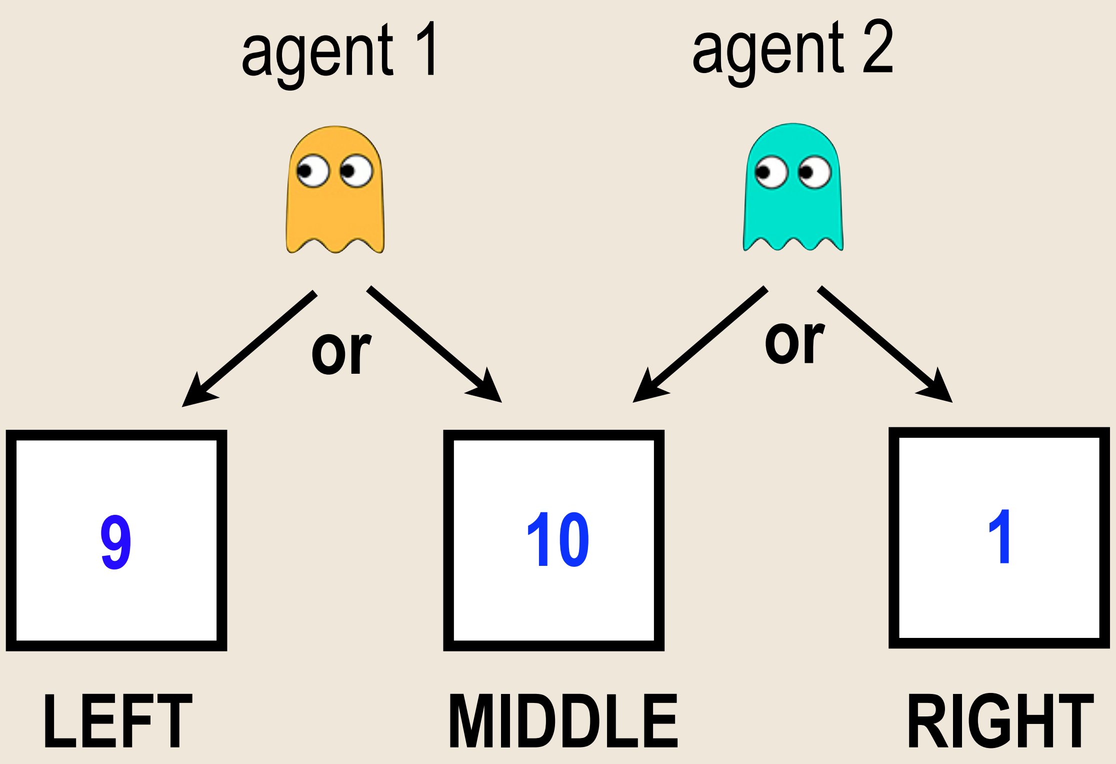
$$\frac{W(\text{greedy algorithm})}{W(\text{best centralized})} \geq \frac{1}{2}$$

- Agents make selections according to order (indices)
- ***Information:*** Available information to each agent $i \Rightarrow x_1, \ldots, x_{i-1}$
- ***Selection rule:*** Maximize marginal contribution given information

$$x_i \in \arg\max_{x_i' \in X_i} W(x_i', x_1, \ldots, x_{i-1}) - W(x_1, \ldots, x_{i-1})$$



Goal Objective

maximize sum of covered values

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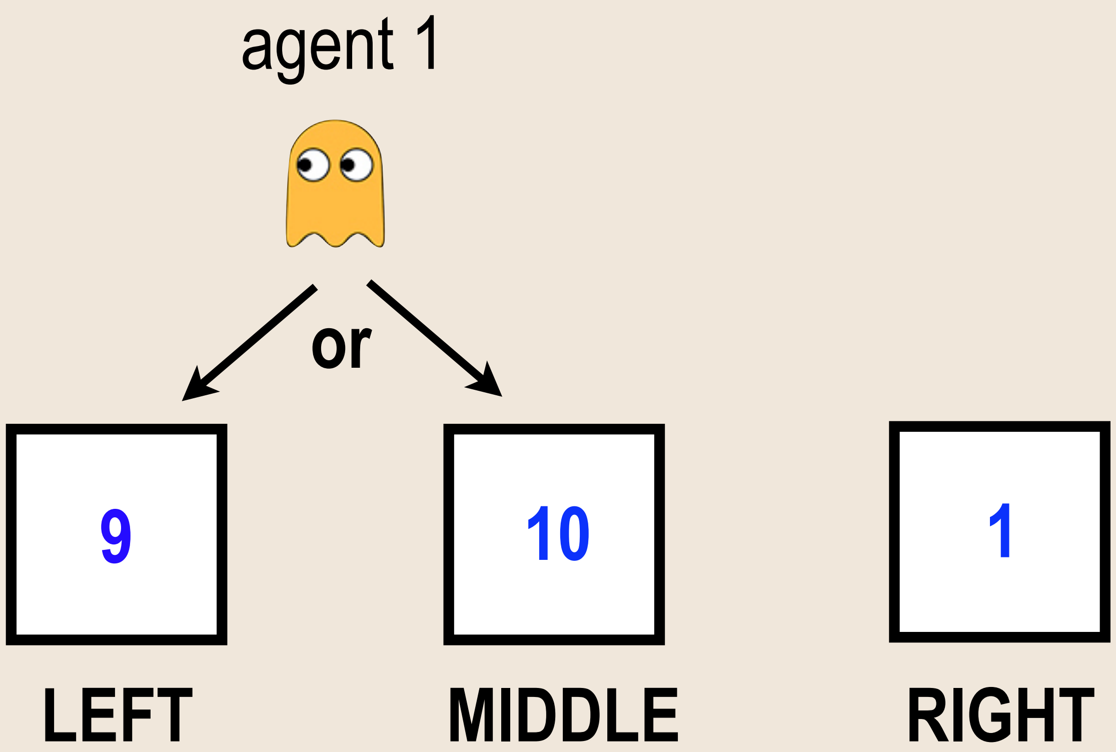
Greedy algorithm



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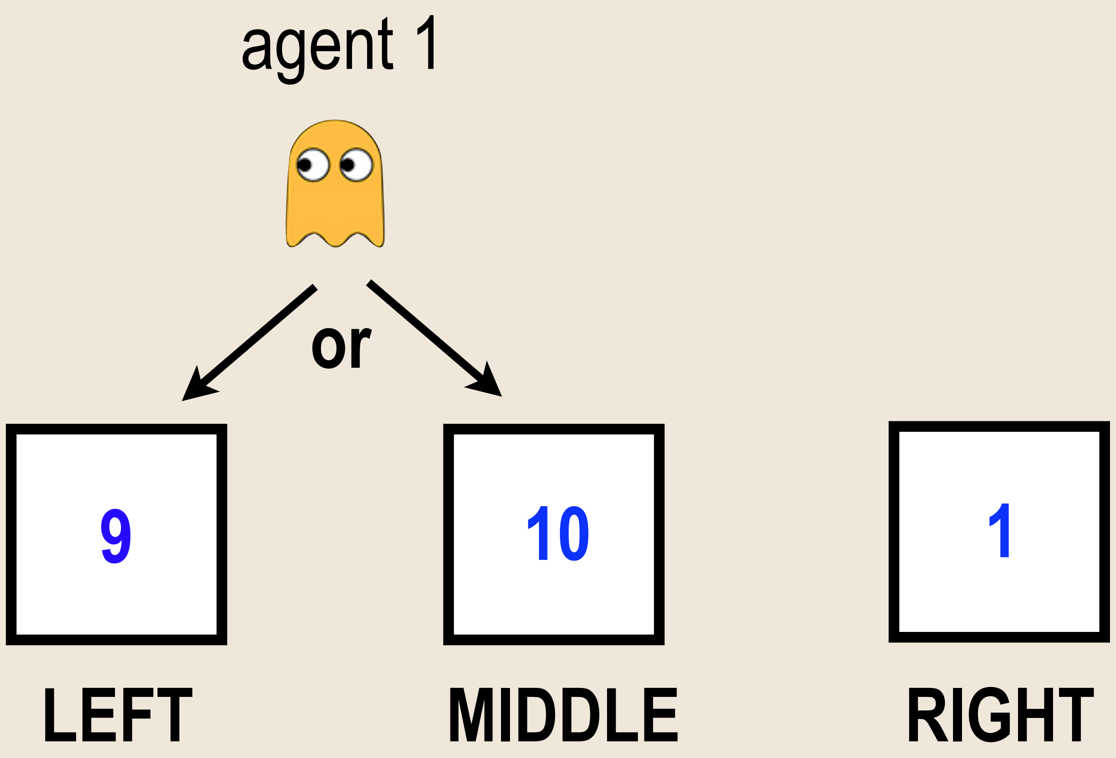

LEFT = 9

MIDDLE = 10

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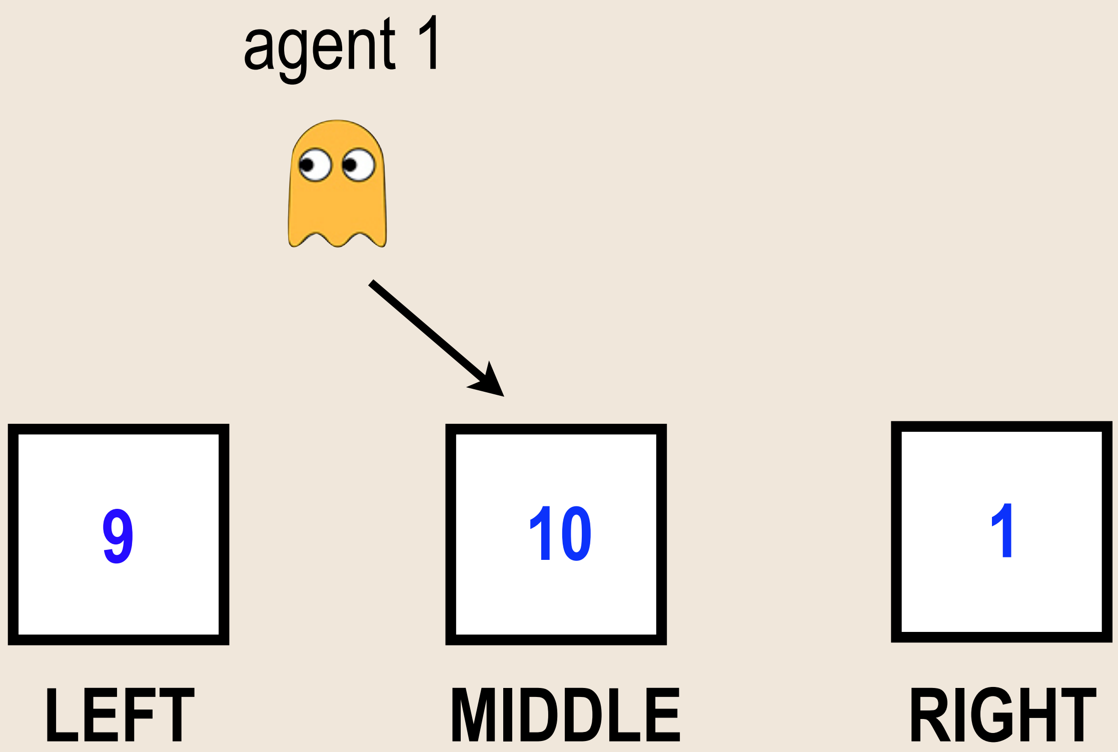

LEFT = 9

MIDDLE = 10

- Agents make selections according to order (indices)
- ***Information:*** Available information to each agent $i \Rightarrow x_1, \ldots, x_{i-1}$
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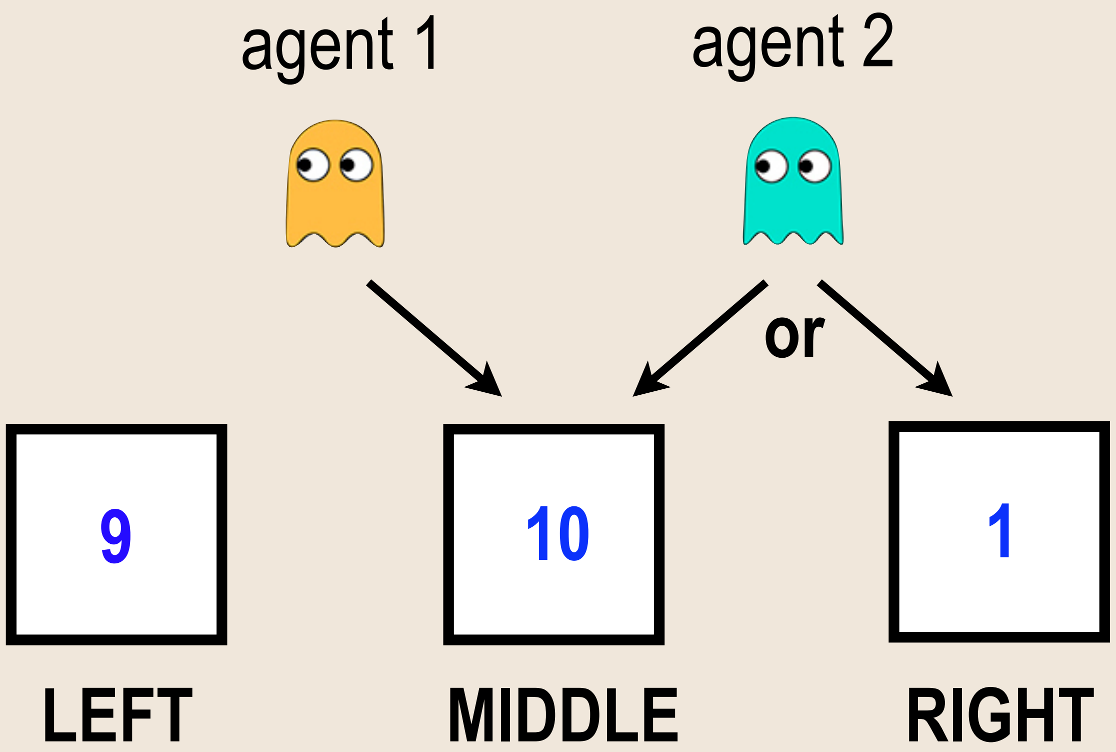

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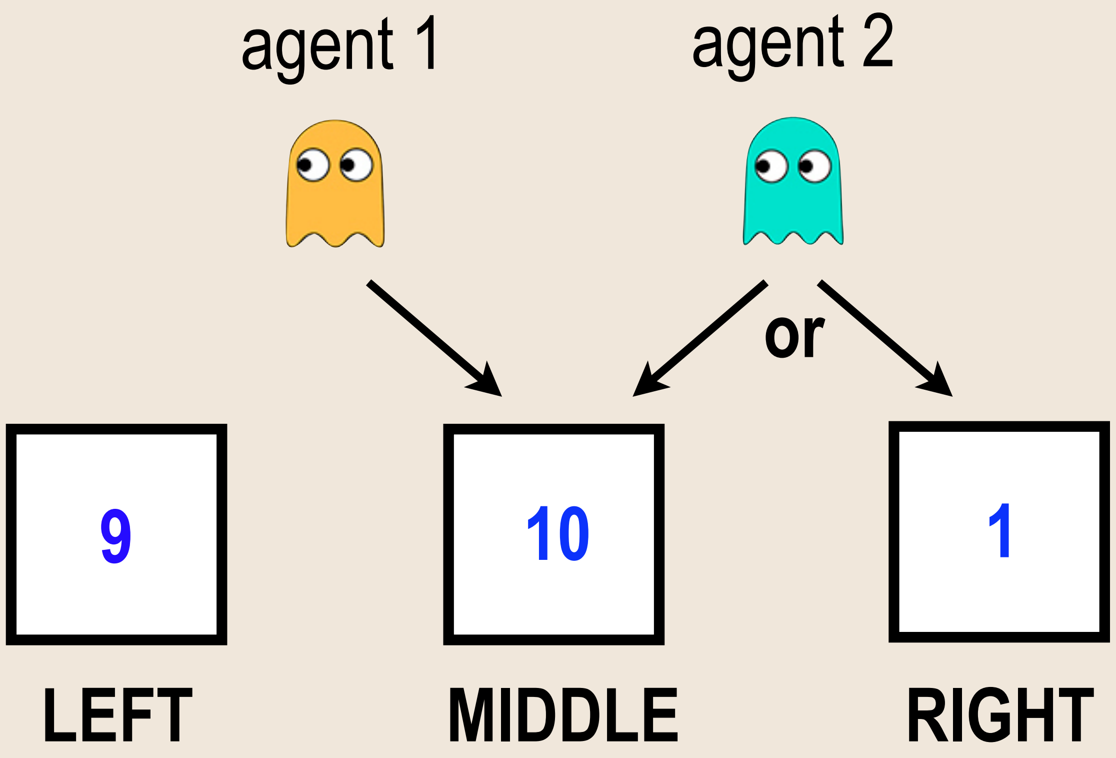
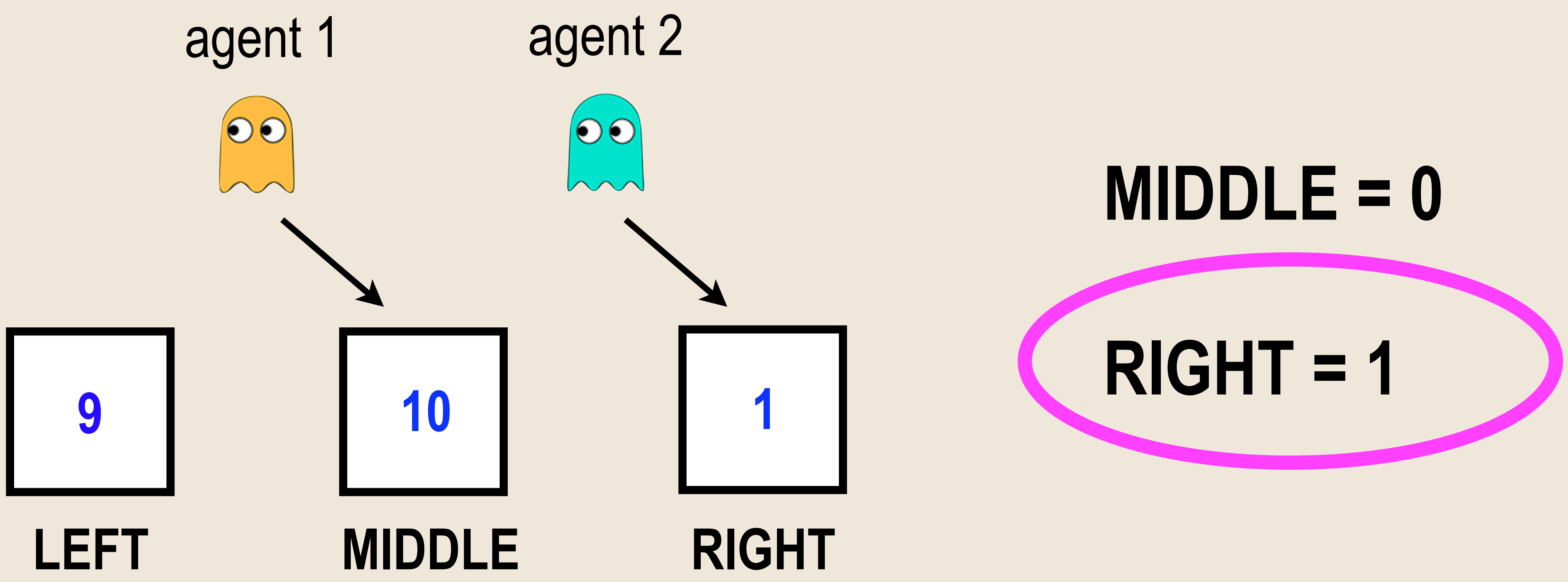


MIDDLE = 0

RIGHT = 1

- Agents make selections according to order (indices)
- ***Information:*** Available information to each agent $i \Rightarrow x_1, \ldots, x_{i-1}$
- ***Selection rule:*** Maximize marginal contribution given information

$$x_i \in \arg\max_{x_i' \in X_i} W(x_i', x_1, \ldots, x_{i-1}) - W(x_1, \ldots, x_{i-1})$$



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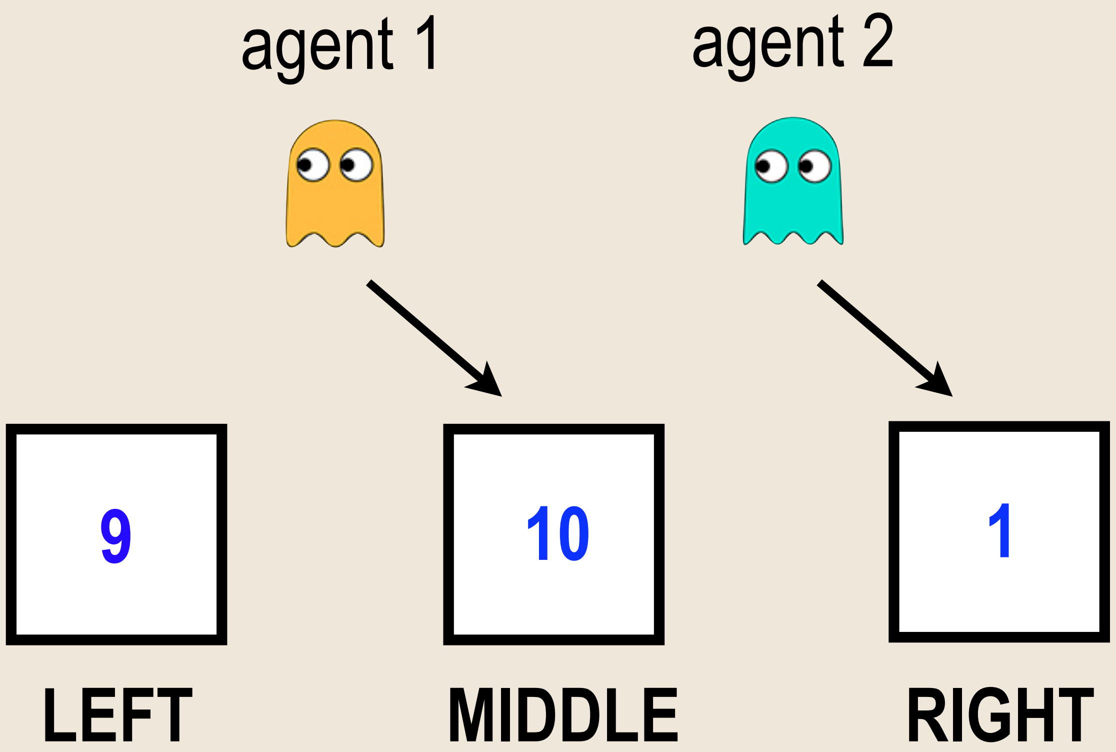
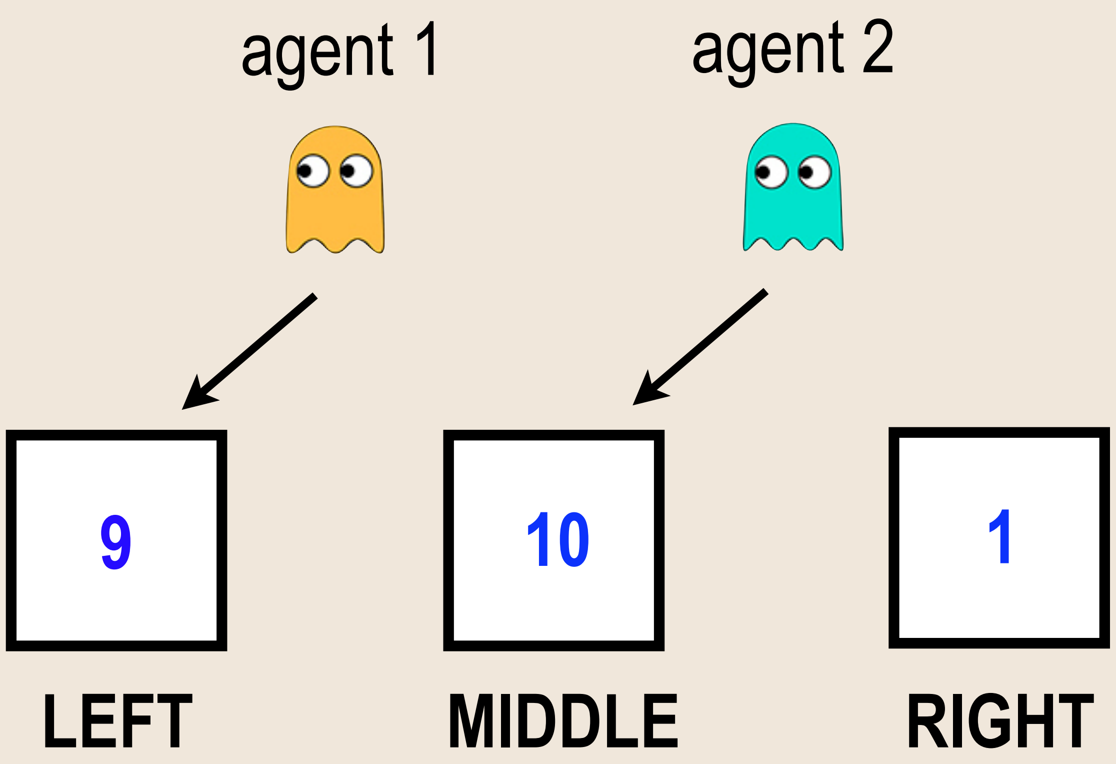


Greedy = 11

- Agents make selections according to order (indices)
- ***Information:*** Available information to each agent $i \Rightarrow x_1, \ldots, x_{i-1}$
- ***Selection rule:*** Maximize marginal contribution given information

$$x_i \in \underset{x_i' \in X_i}{\arg\max}\; W(x_i', x_1, \ldots, x_{i-1}) - W(x_1, \ldots, x_{i-1})$$



Greedy = 11

Optimal = 19

- Agents make selections according to order (indices)
- ***Information:*** Available information to each agent $i \Rightarrow x_1, \ldots, x_{i-1}$
- ***Selection rule:*** Maximize marginal contribution given information

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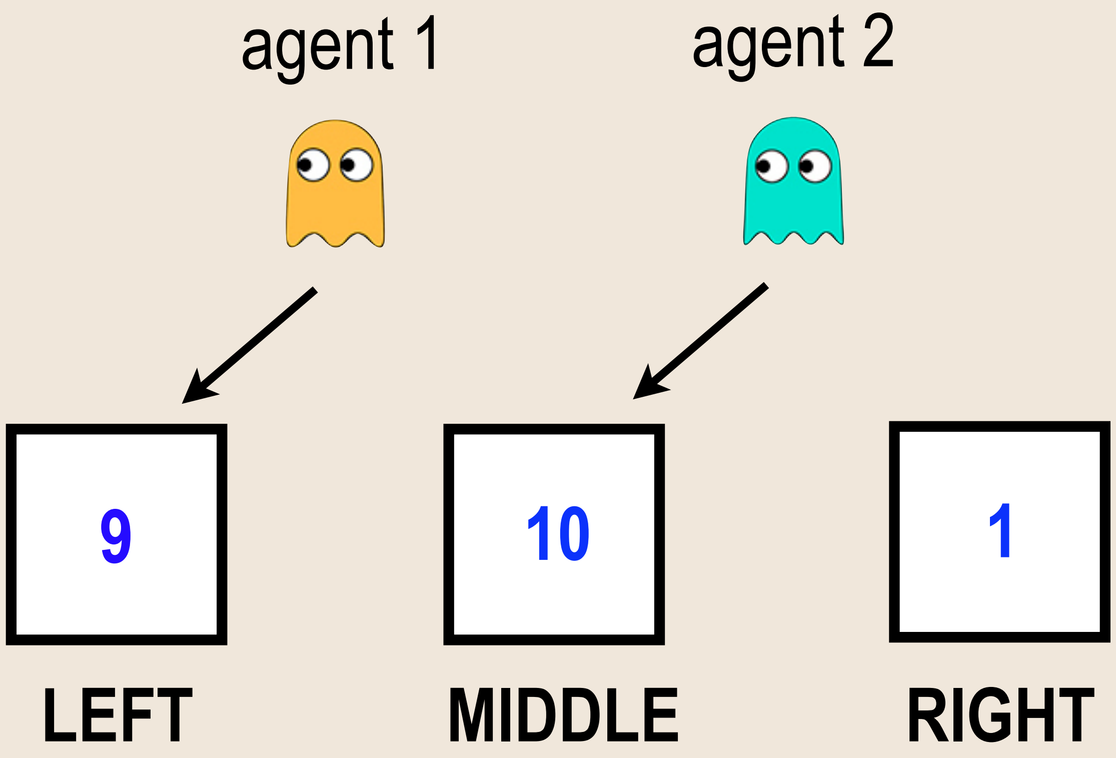

Greedy = 11

Optimal = 19

$$\frac{W(\text{greedy algorithm})}{W(\text{best centralized})} = \frac{11}{19} \geq \frac{1}{2}$$

bound holds irrespective of number agents, assigned order, boxes, values, action sets, etc

Greedy Algorithm

- Agents make selections according to order (indices)
- ***Information:*** Available information to each agent $i \Rightarrow x_1, \ldots, x_{i-1}$
- ***Selection rule:*** Maximize marginal contribution given information

$$x_i \in \arg\max_{x_i' \in X_i} W(x_i', x_1, \ldots, x_{i-1}) - W(x_1, \ldots, x_{i-1})$$

Greedy Algorithm

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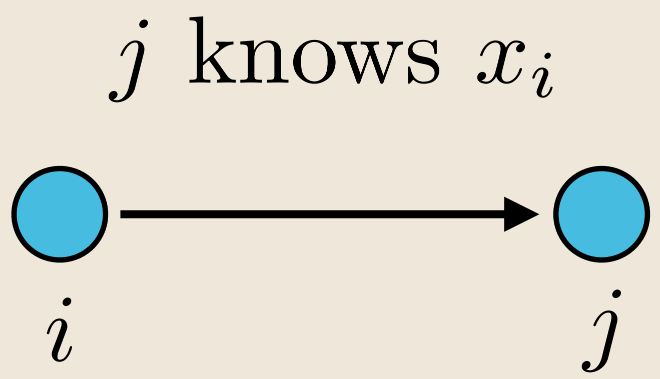
Questions:

- What happens if agents do not have all information needed?

Information Graph G

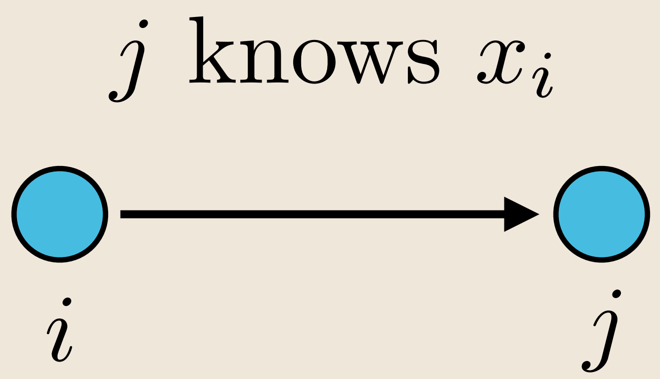
- Nodes: Agents
- Edges: Informational availability

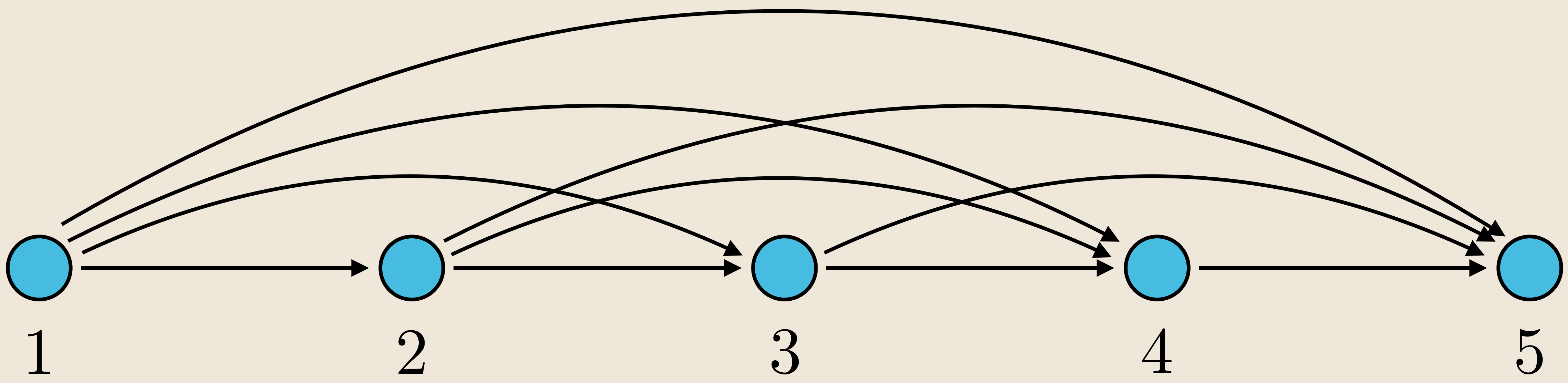


B. Gharesifard and S. L. Smith. "Distributed submodular maximization with limited information," 2017

Information Graph G

- Nodes: Agents
- Edges: Informational availability





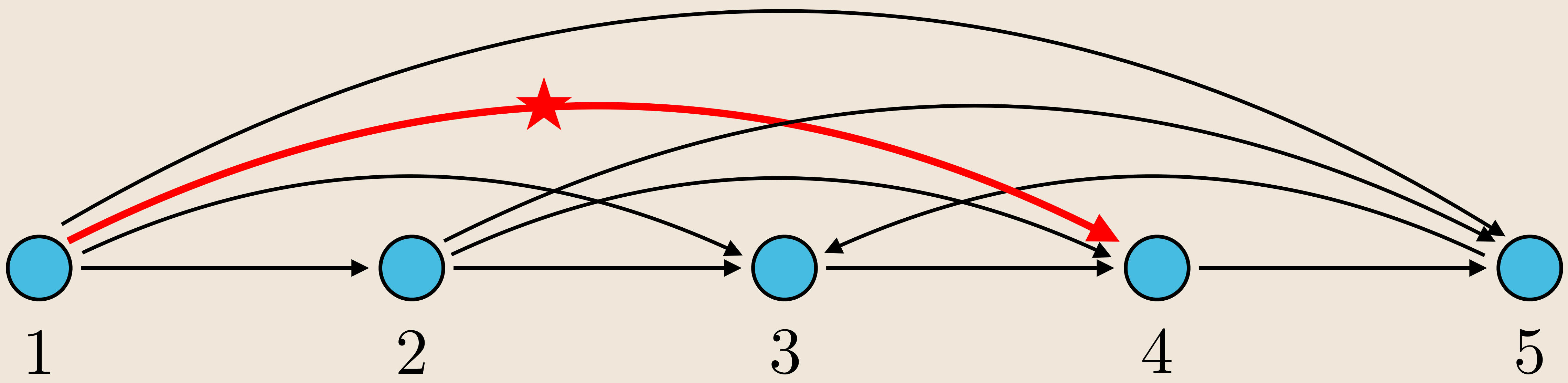
information graph for standard greedy algorithm

B. Gharesifard and S. L. Smith. "Distributed submodular maximization with limited information," 2017

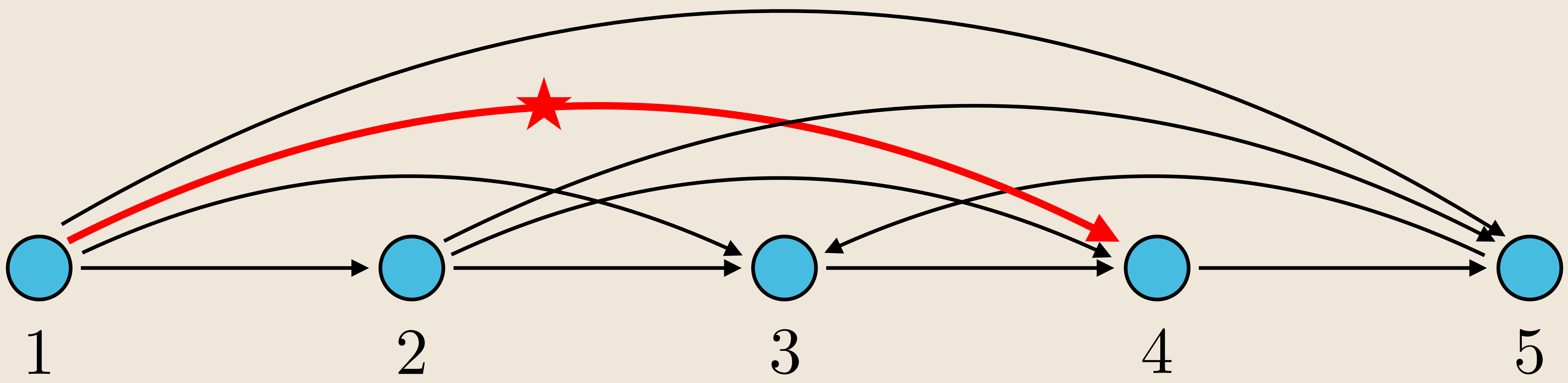
Information Graph G

- Nodes: Agents
- Edges: Informational availability

j knows x_i

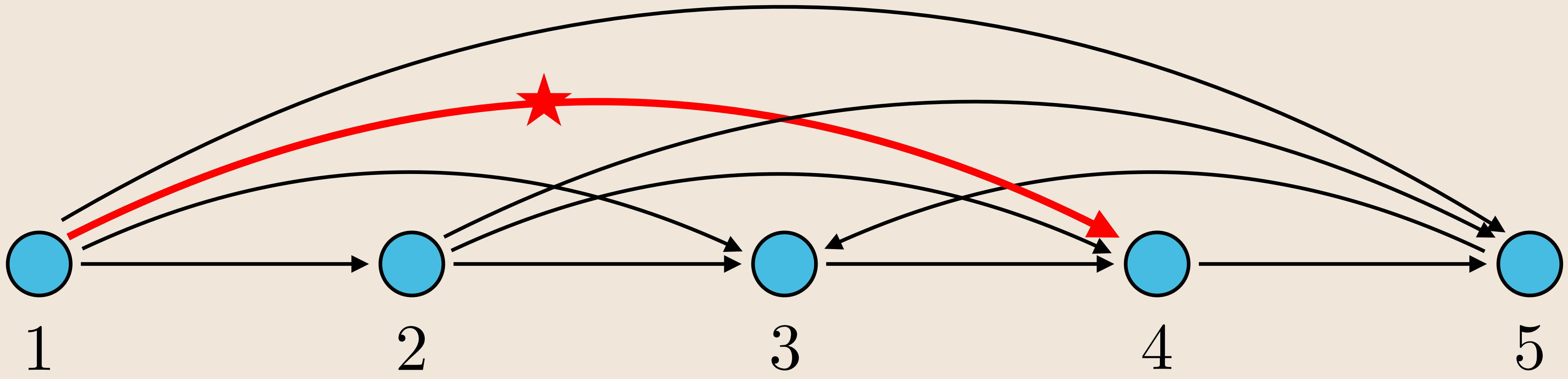


$$x_1 \in \arg\max_{x_1' \in X_1} W(x_1')$$



$$x_1 \in \arg\max_{x'_1 \in X_1} W(x'_1)$$

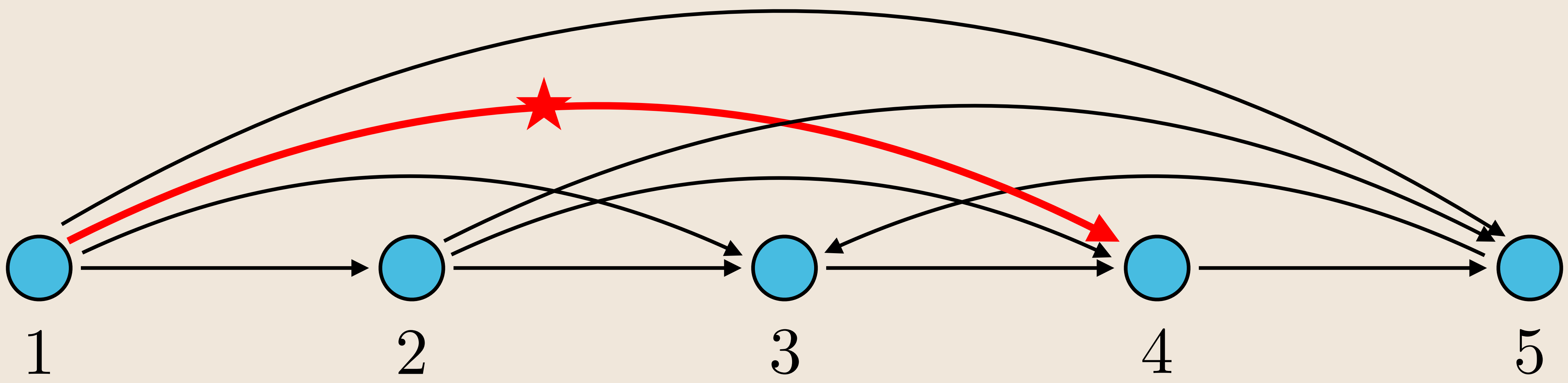
$$x_2 \in \arg\max_{x'_2 \in X_2} W(x'_2, x_1) - W(x_1)$$



$$x_1 \in \arg\max_{x_1' \in X_1} W(x_1')$$

$$x_2 \in \arg\max_{x_2' \in X_2} W(x_2', x_1) - W(x_1)$$

$$x_3 \in \arg\max_{x_3' \in X_3} W(x_3', x_1, x_2) - W(x_1, x_2)$$

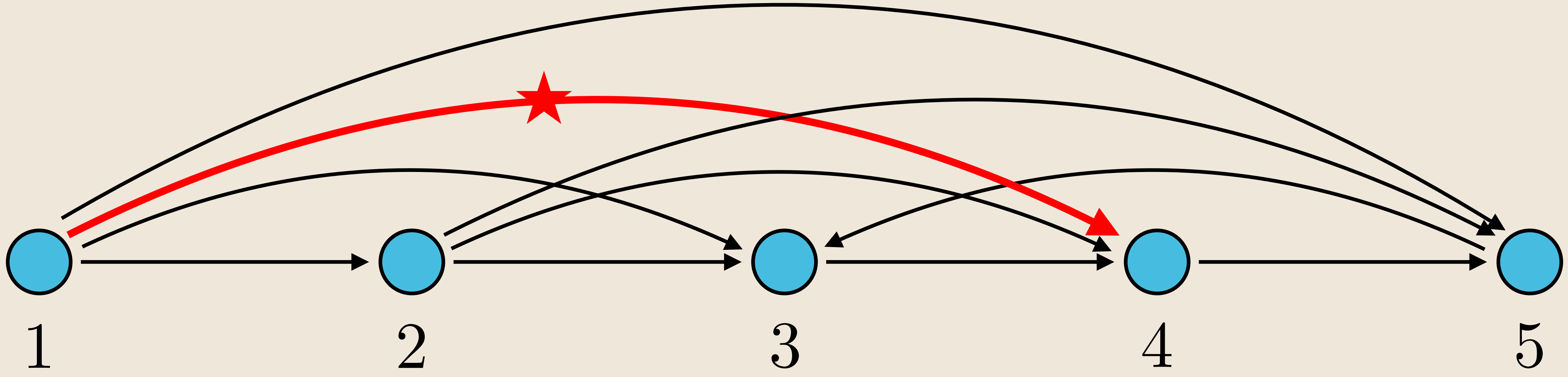


$$x_1 \in \arg\max_{x_1' \in X_1} W(x_1')$$

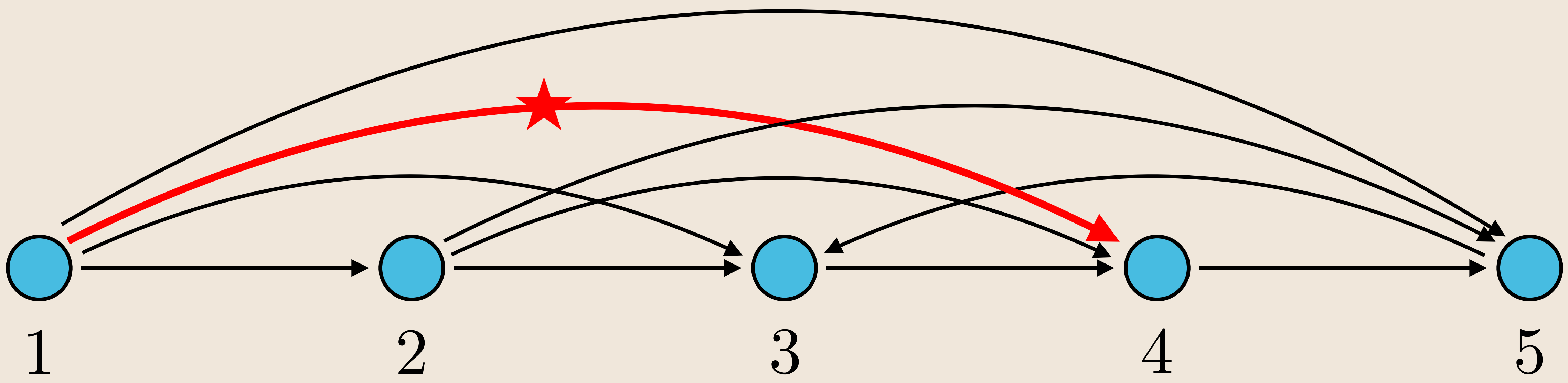
$$x_2 \in \arg\max_{x_2' \in X_2} W(x_2', x_1) - W(x_1)$$

$$x_3 \in \arg\max_{x_3' \in X_3} W(x_3', x_1, x_2) - W(x_1, x_2)$$

$$x_4 \in \arg\max_{x_4' \in X_4} W(x_4', x_2, x_3) - W(x_2, x_3)$$ (does not have access to x_1)



How does the structure of the information graph impact the quality of the localized greedy solution?



Theorem [Grimsman et al., 2018]

Consider any submodular multiagent optimization problem with an information graph G. The quality of the localized greedy solution satisfies

$$\frac{W(x^{\text{greedy}}; G)}{W(x^{\text{optimal}})} \geq \frac{1}{1 + \alpha^*(G)}$$

where $\alpha^*(G)$ is the (fractional) independence number of the graph G. Furthermore, the bound is essentially tight.

Grimsman et al., "Value of Information in Greedy Submodular Maximization," TCNS, 2019.

Independent set: An independent set is a set of vertices *J* such that

$$v_1, v_2 \in J \;\Rightarrow (v_1, v_2), (v_2, v_1) \notin E$$

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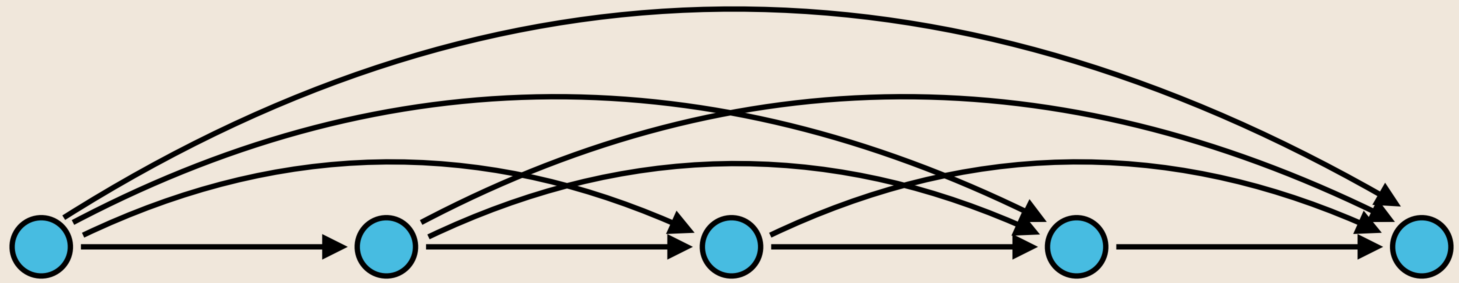
$$\alpha(G) = 1$$



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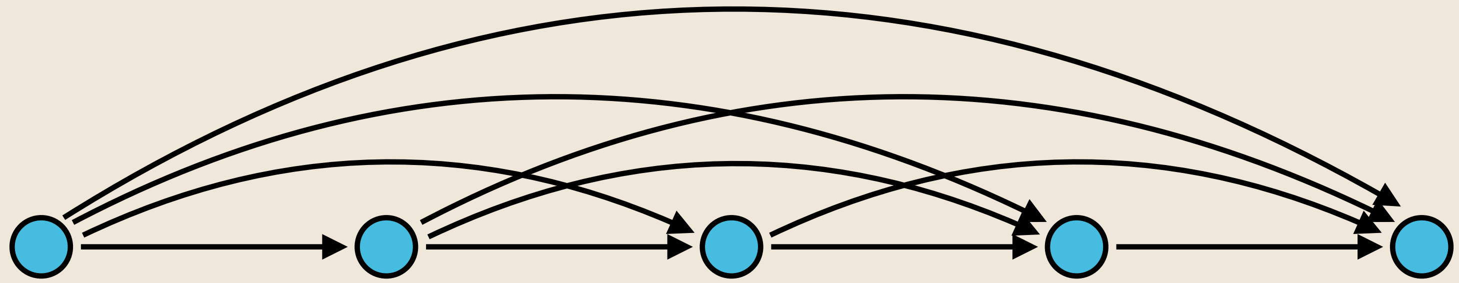
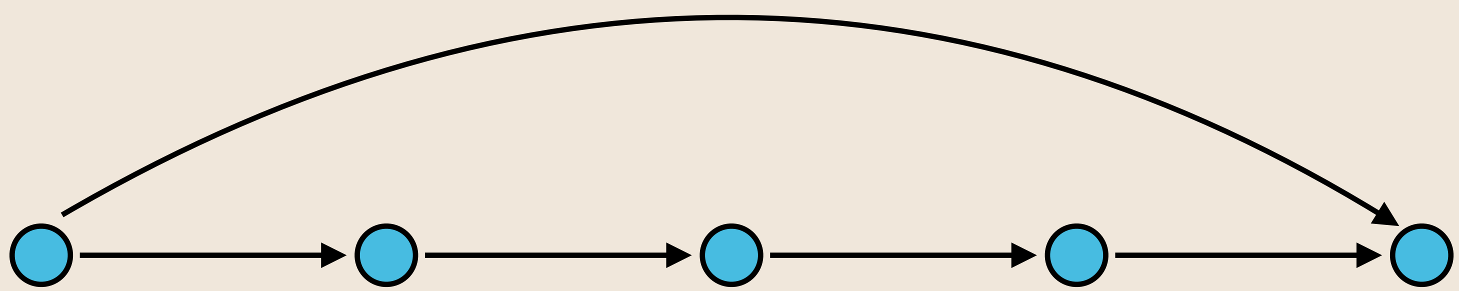


$\alpha(G) = 2$



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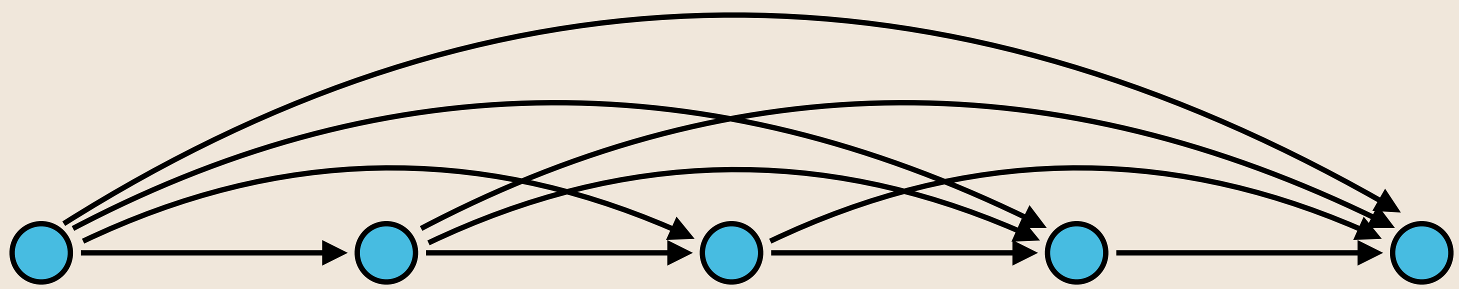
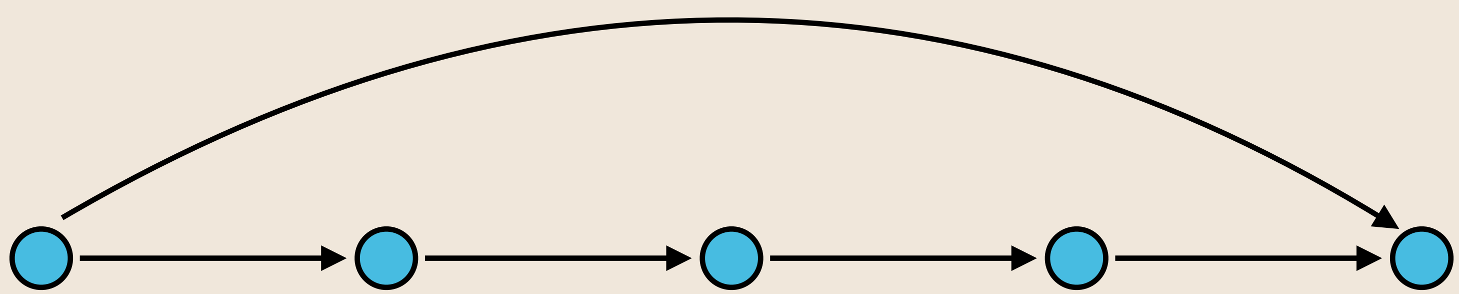
$\alpha(G) = 1$



$\alpha(G) = 2$



Fractional independence number: Real number relaxation (LP)

Grimsman et al., “Value of Information in Greedy Submodular Maximization,” TCNS, 2019.

Main result

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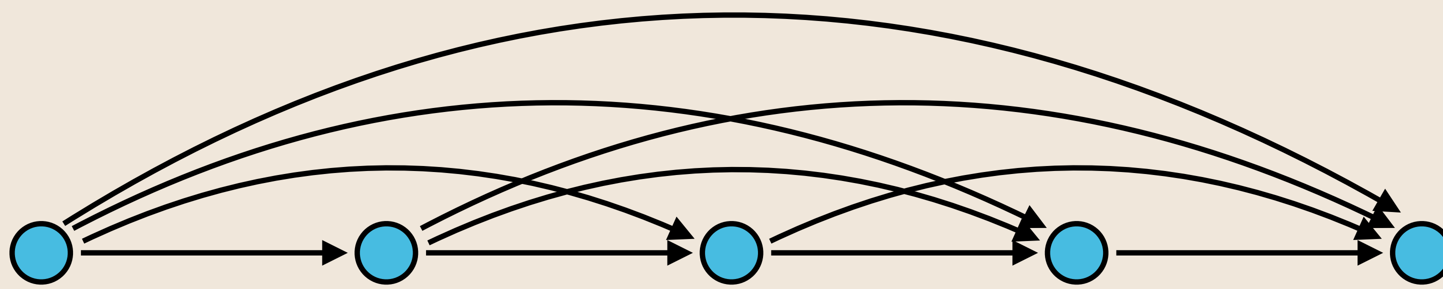
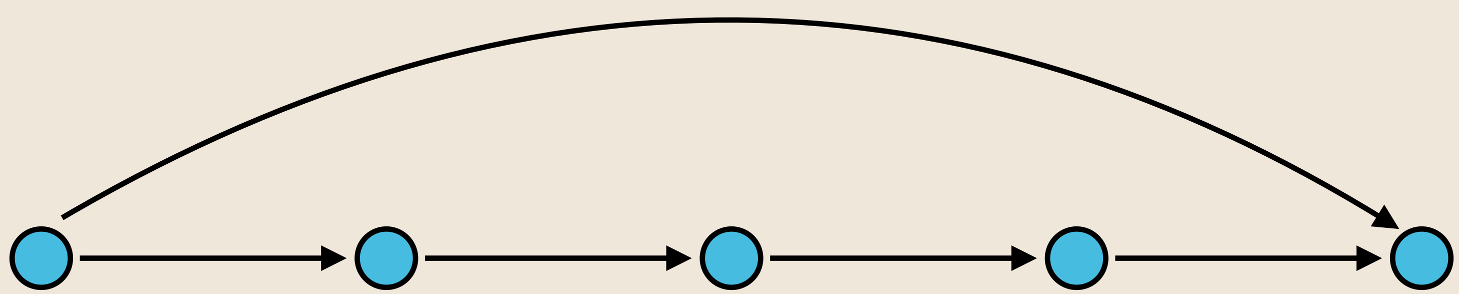


$\alpha^*(G) = 1$

$\alpha(G) = 2$



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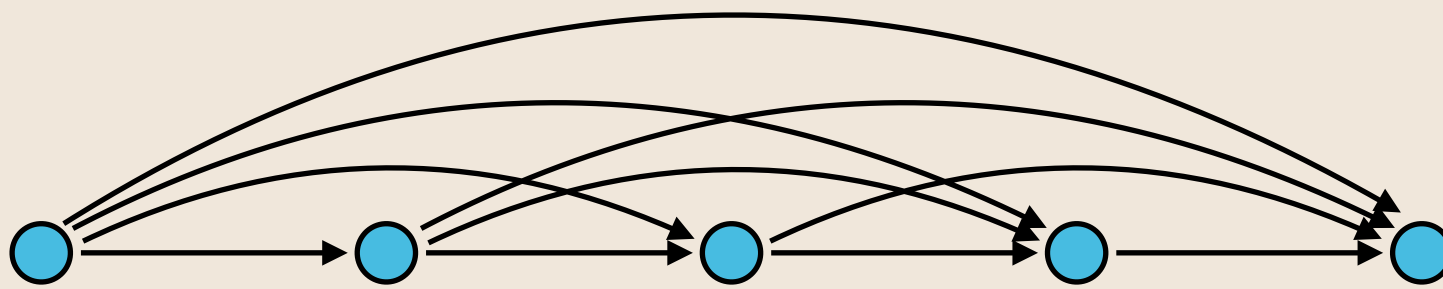
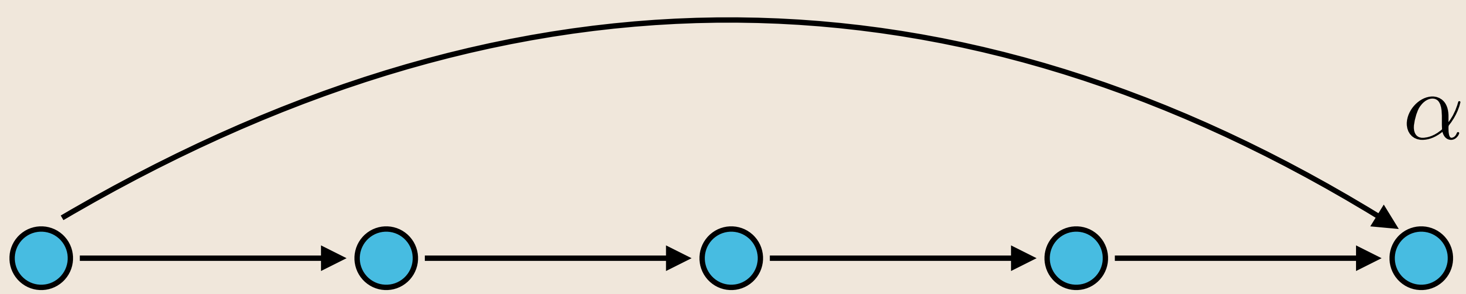
Independence number: Cardinality of largest independent set

$\alpha(G) = 1$ $\qquad$ $\alpha^*(G) = 1$



$\alpha(G) = 2$ $\qquad$ $\alpha^*(G) = 2.5$



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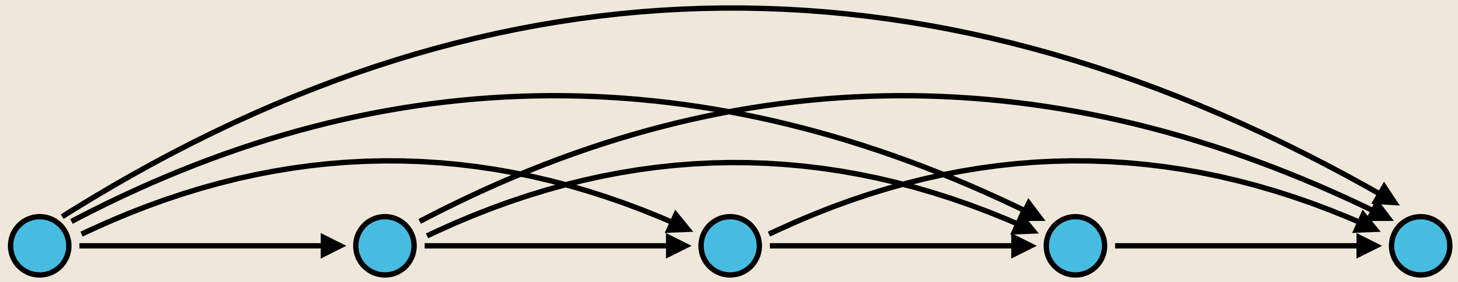
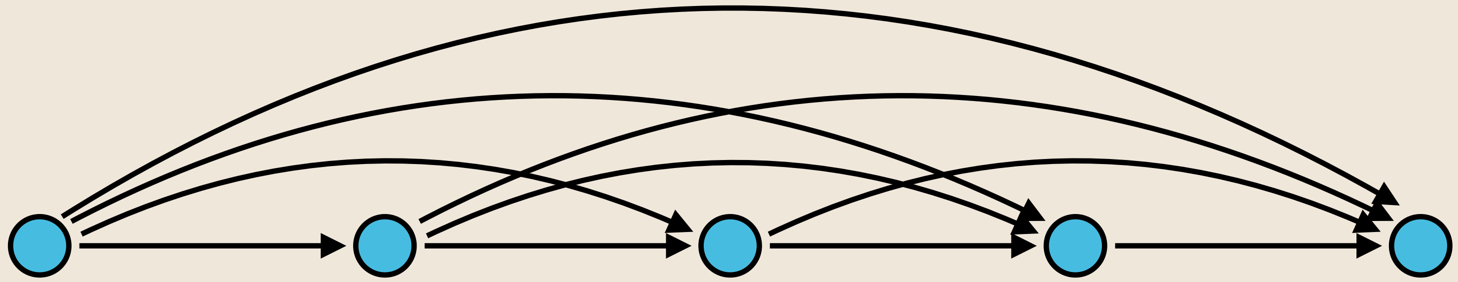
Main result



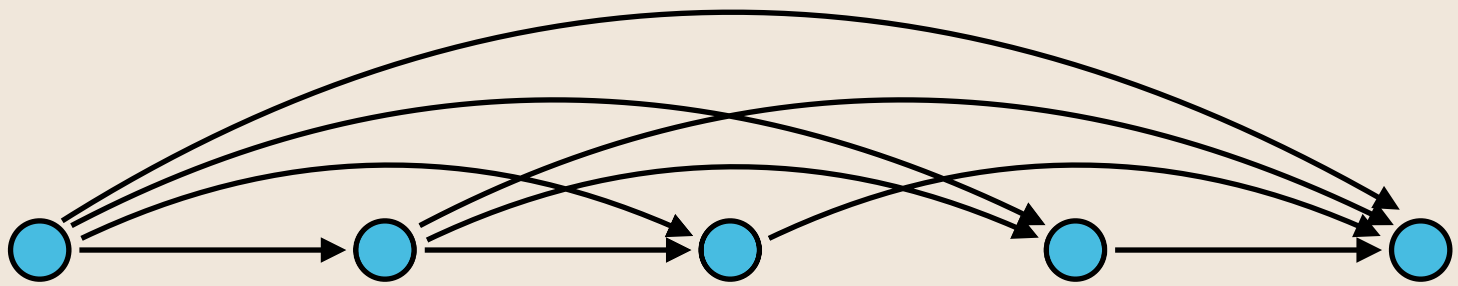
$$\frac{W(x^{\text{greedy}}; G)}{W(x^{\text{optimal}})} \geq \frac{1}{2}$$

Grimsman et al., “Value of Information in Greedy Submodular Maximization,” TCNS, 2019.

Main result

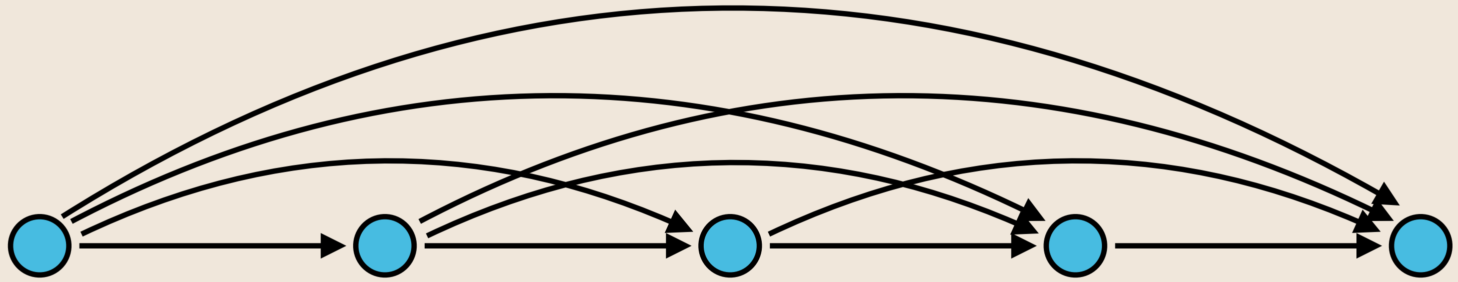


$$\frac{W(x^{\text{greedy}}; G)}{W(x^{\text{optimal}})} \geq \frac{1}{2}$$



$$\frac{W(x^{\text{greedy}}; G)}{W(x^{\text{optimal}})} \geq \frac{1}{3}$$

Grimsman et al., "Value of Information in Greedy Submodular Maximization," TCNS, 2019.



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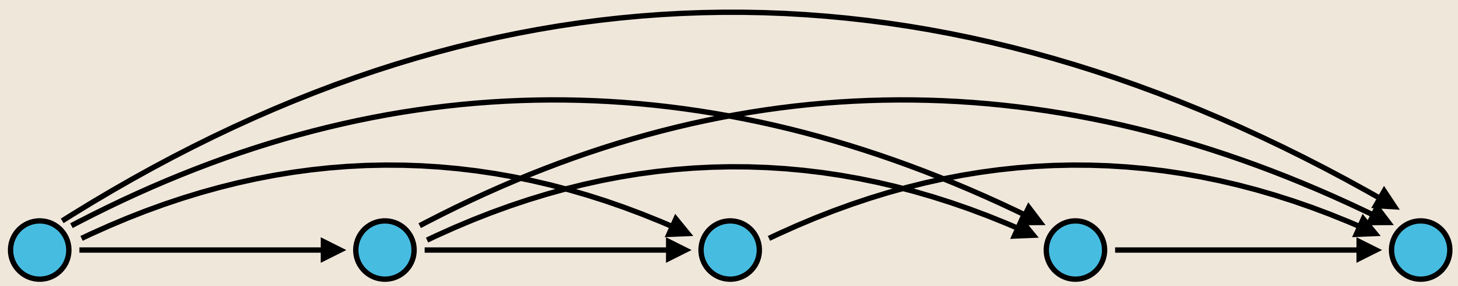

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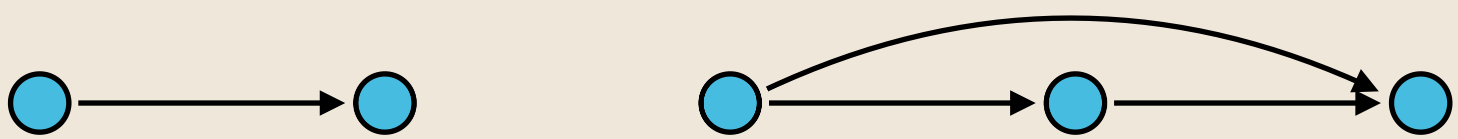

$$\frac{W(x^{\text{greedy}}; G)}{W(x^{\text{optimal}})} \geq \frac{1}{3}$$

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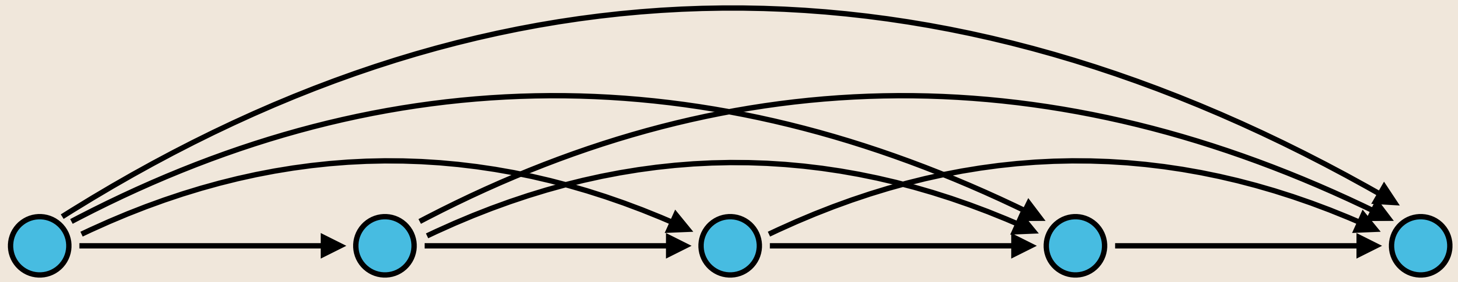
Main result



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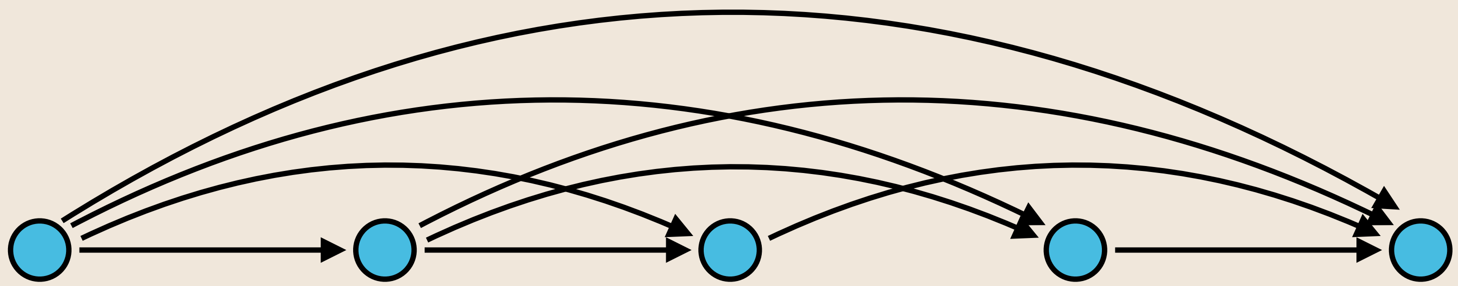

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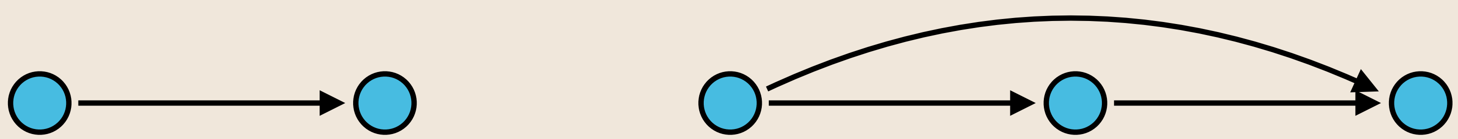

$$\frac{W(x^{\text{greedy}}; G)}{W(x^{\text{optimal}})} \geq \frac{1}{3}$$

$$\frac{W(x^{\text{greedy}}; G)}{W(x^{\text{optimal}})} \geq \frac{1}{4}$$

Grimsman et al., "Value of Information in Greedy Submodular Maximization," TCNS, 2019.

Greedy Algorithm

- Agents make selections according to order (indices)
- ***Information:*** Available information to each agent $i \Rightarrow x_1, \ldots, x_{i-1}$
- ***Selection rule:*** Maximize marginal contribution given information

$$x_i \in \arg\max_{x_i' \in X_i} W(x_i', x_1, \ldots, x_{i-1}) - W(x_1, \ldots, x_{i-1})$$

Questions:

- What happens if agents do not have all information needed?

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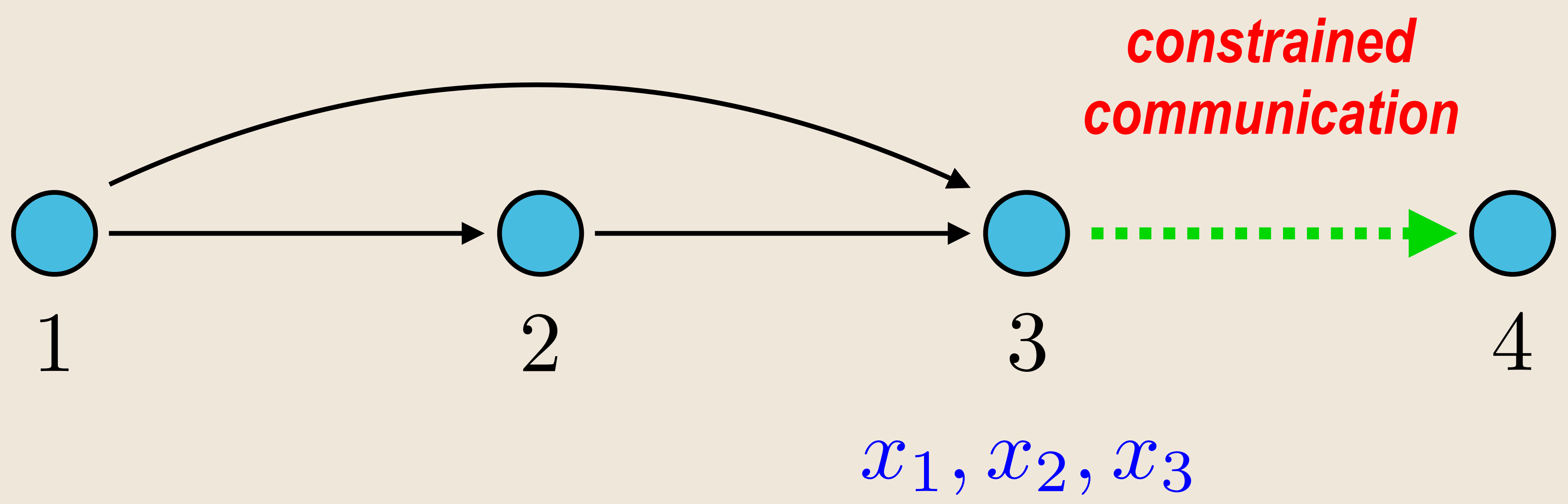
Questions:

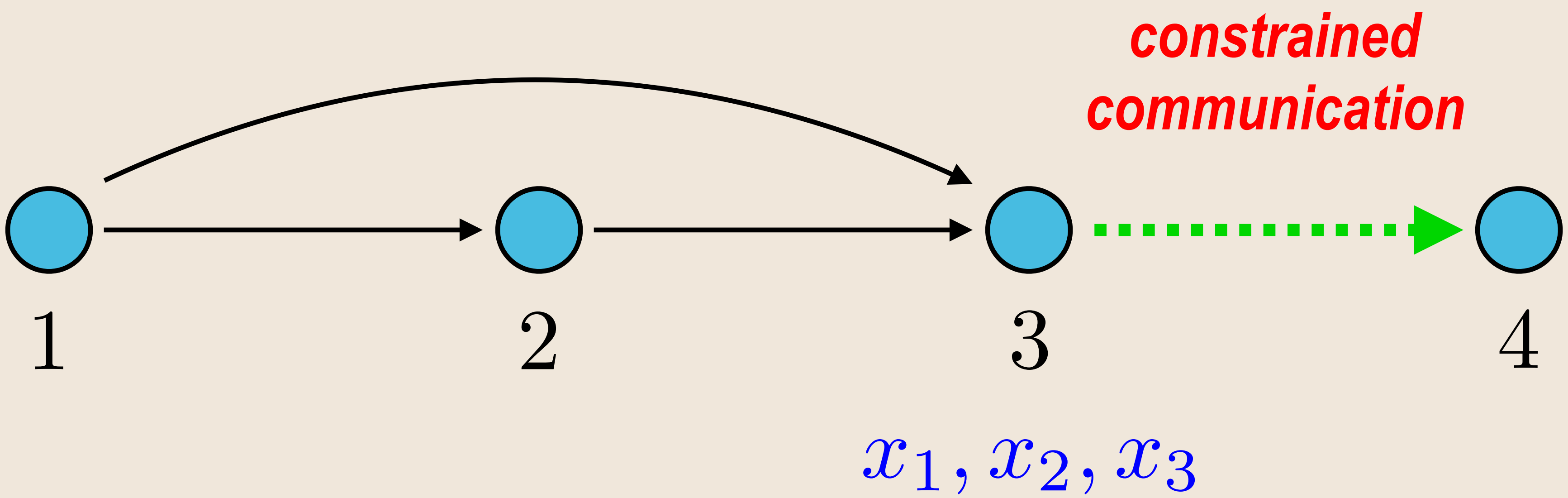
- What happens if agents do not have all information needed?
- How do you capitalize off strategic information exchange?



can only transmit x_1 or x_2 or x_3

Grimsman et al., “Strategic Information Sharing in Greedy Submodular Maximization,” CDC, 2018.

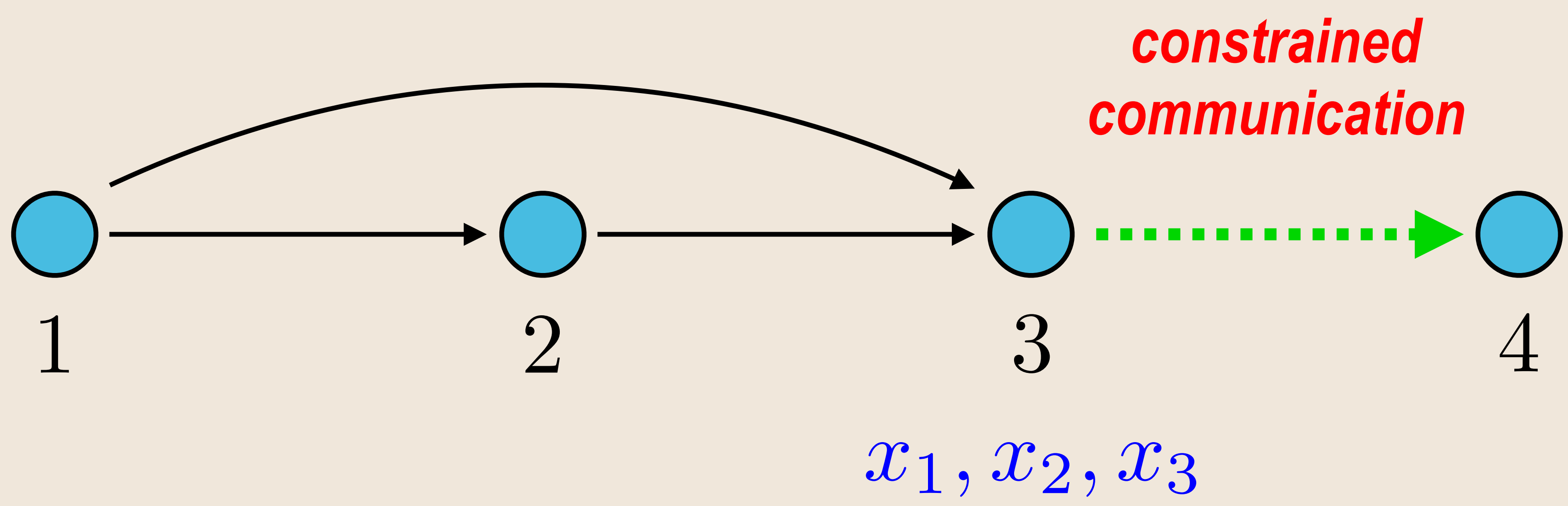
Strategic Information Exchange



can only transmit x_1 or x_2 or x_3

What information should agent 3 transmit to agent 4?

Grimsman et al., “Strategic Information Sharing in Greedy Submodular Maximization,” CDC, 2018.



can only transmit x_1 or x_2 or x_3

What information should agent 3 transmit to agent 4?

How should agent 4 utilize the transmitted information?

Grimsman et al., "Strategic Information Sharing in Greedy Submodular Maximization," CDC, 2018.

Greedy Algorithm

- Agents make selections according to order (indices)
- ***Information:*** Available information to each agent $i \Rightarrow x_1, \ldots, x_{i-1}$
- ***Selection rule:*** Maximize marginal contribution given information

$$x_i \in \arg\max_{x_i' \in X_i} W(x_i', x_1, \ldots, x_{i-1}) - W(x_1, \ldots, x_{i-1})$$

Questions:

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$$x_i \in \arg\max_{x_i' \in X_i} W(x_i', x_1, \ldots, x_{i-1}) - W(x_1, \ldots, x_{i-1})$$

Questions:

- What happens if agents do not have all information needed?
- How do you capitalize off strategic information exchange?
- Can alternative selection rules yield improved performance guarantees?

Greedy Algorithm

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- ***Information:*** Available information to each agent $i \Rightarrow x_1, \ldots, x_{i-1}$
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$$x_i \in \arg\max_{x_i' \in X_i} W(x_i', x_1, \ldots, x_{i-1}) - W(x_1, \ldots, x_{i-1})$$

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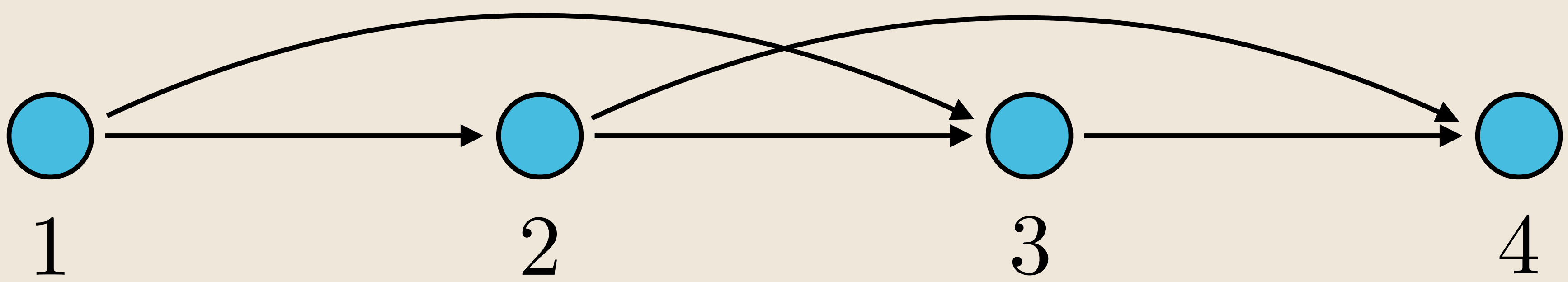


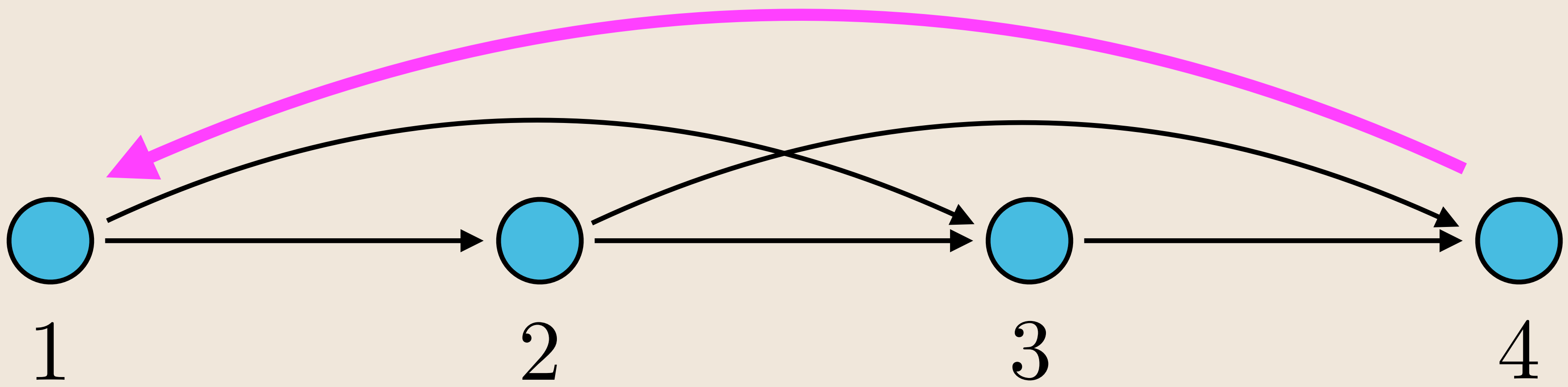
acyclic graphs

optimal selection rule

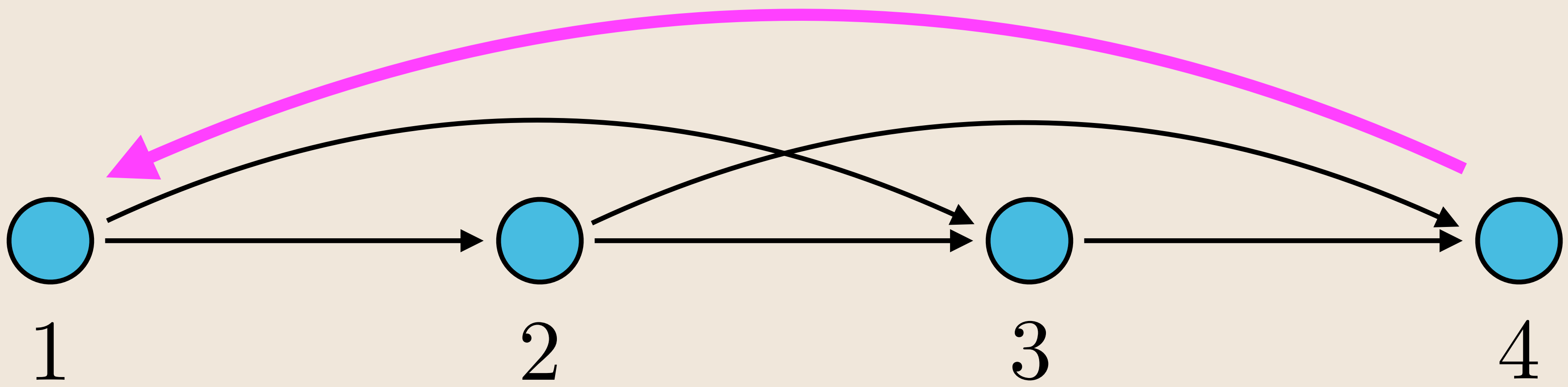
Selection rule: Maximize marginal contribution given information

$$x_i \in \arg\max_{x_i' \in X_i} W(x_i', x_1, \ldots, x_{i-1}) - W(x_1, \ldots, x_{i-1})$$

(equivalent to maximizing system-level objectives given information)



cyclic graphs
(rest point of round robin greedy)



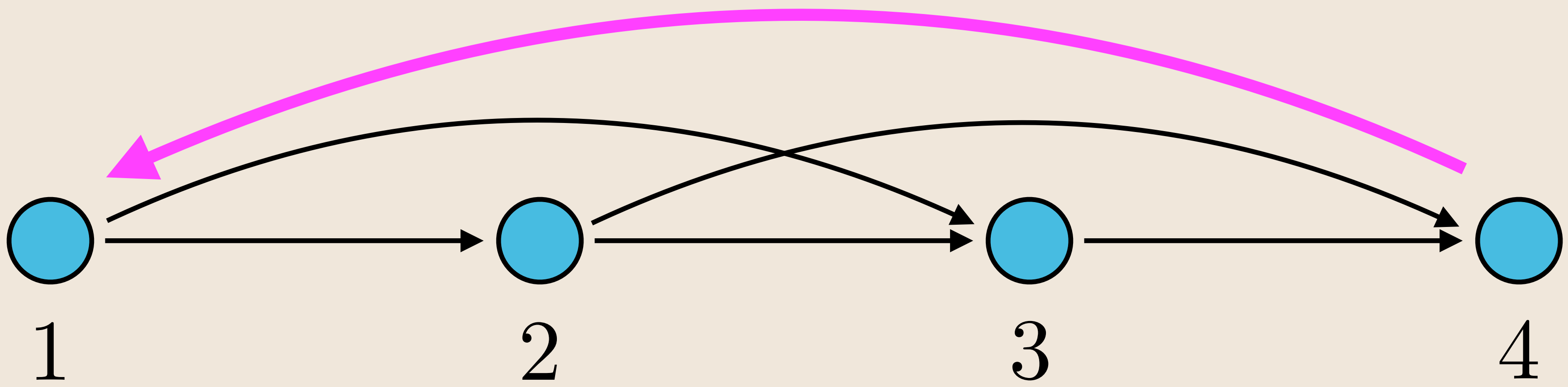
cyclic graphs
(rest point of round robin greedy)

no longer optimal selection rule

Selection rule: Maximize marginal contribution given information

$$x_i \in \arg\max_{x_i' \in X_i} W(x_i', x_1, \ldots, x_{i-1}) - W(x_1, \ldots, x_{i-1})$$

Agents optimizing global objective ***NOT*** an optimal strategy!!!!



cyclic graphs
(rest point of round robin greedy)

performance gains >30% by switching selection rule

Ramaswamy et al., “Multiagent Coverage Problems:
The Trade-offs Between Anarchy and Stability,” 2019 (in review).

Two main components:

Transcription

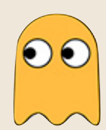
Policy generation

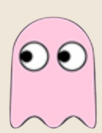
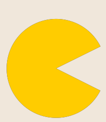
Part I

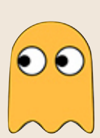
How does lack of information degrade achievable performance?

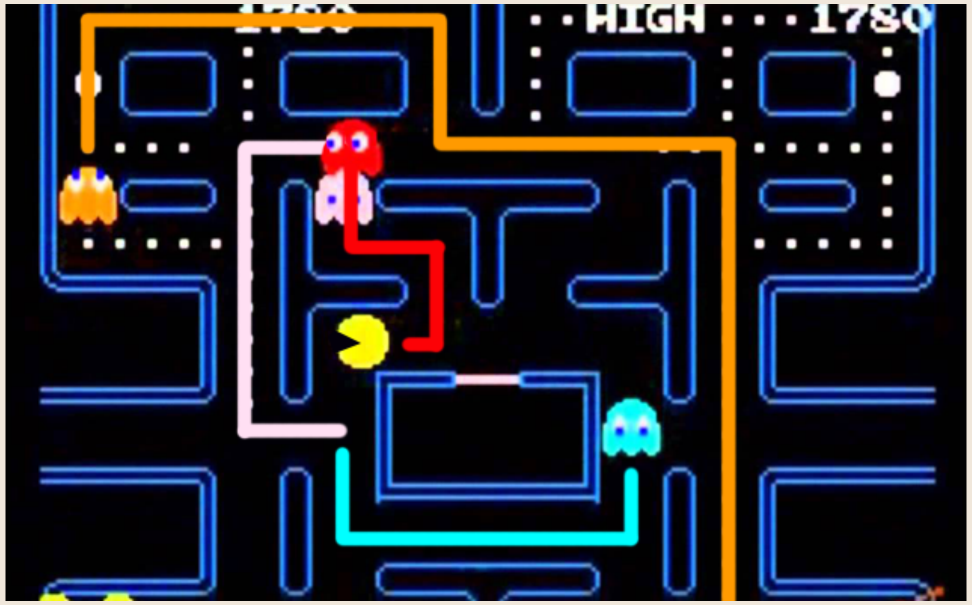
Part II

How do you optimize collective performance using available information?

What information does  have?

- , ,  planned paths?
-  location?
- localized board info?

What should  do with this information?



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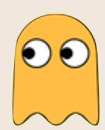
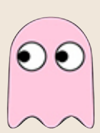
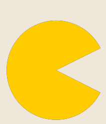
What information does  have?

- , ,  planned paths?
-  location?
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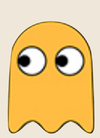
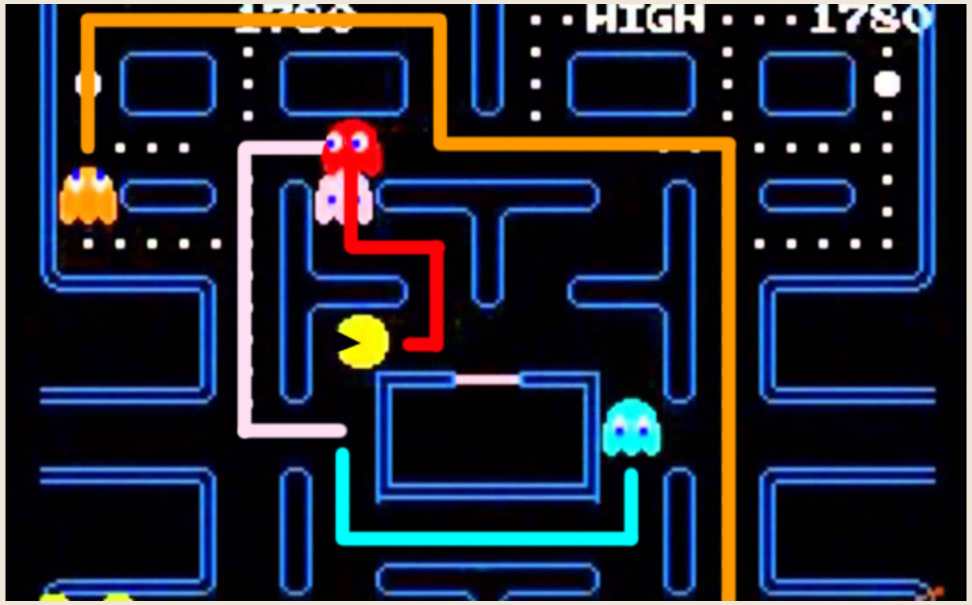
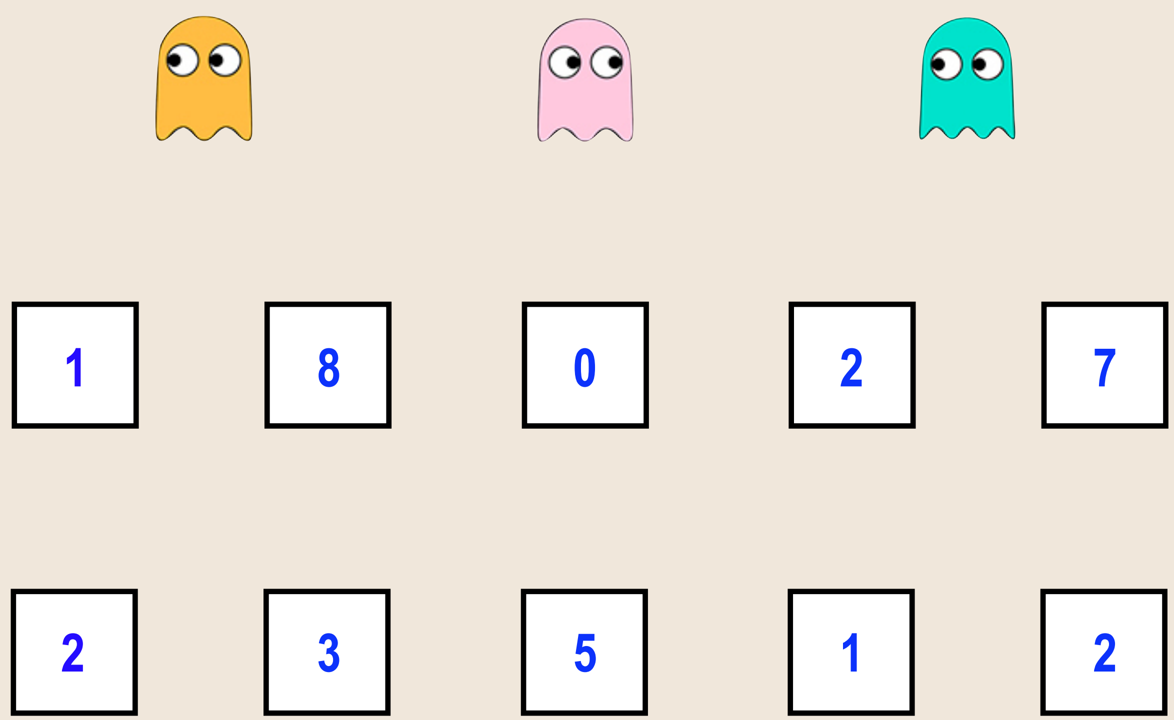
What should  do with this information?



Setup:

- Resources: $\mathcal{R}$
- Values: $v_r \geq 0$
- Actions: $X_i \subseteq 2^{\mathcal{R}}, \; i \in N$
- Global Welfare: $W(x) = \sum_{r \in \cup x_i} v_r$



M. Gairing, “Covering Games: Approximations through Non-Cooperation”, 2009

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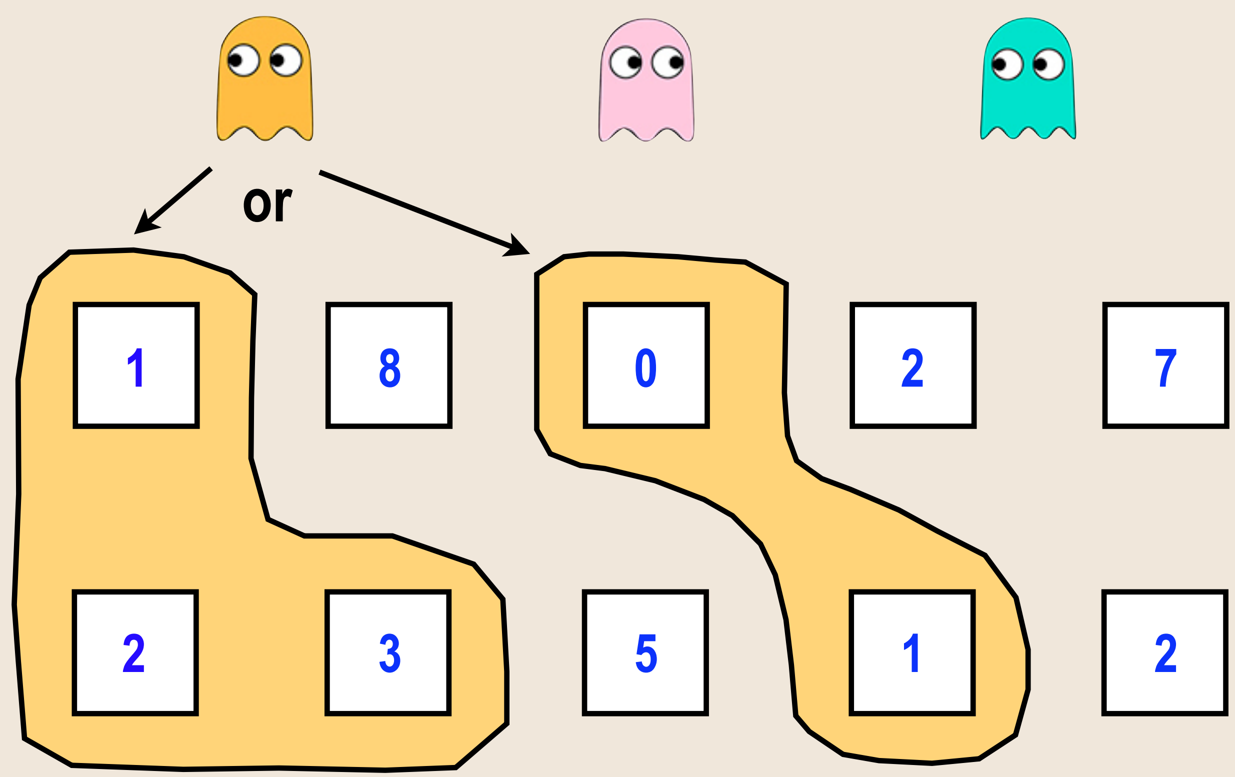

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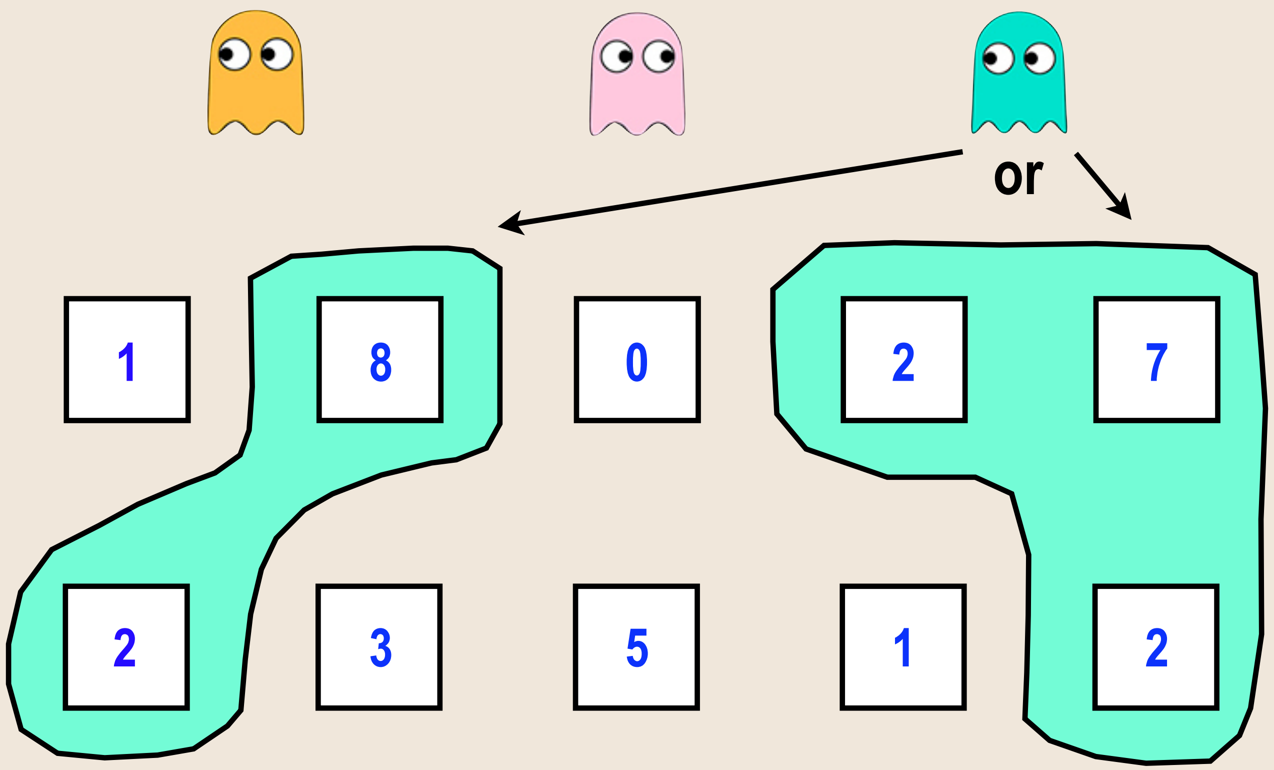

Setup:

- Resources: $\mathcal{R}$
- Values: $v_r \geq 0$
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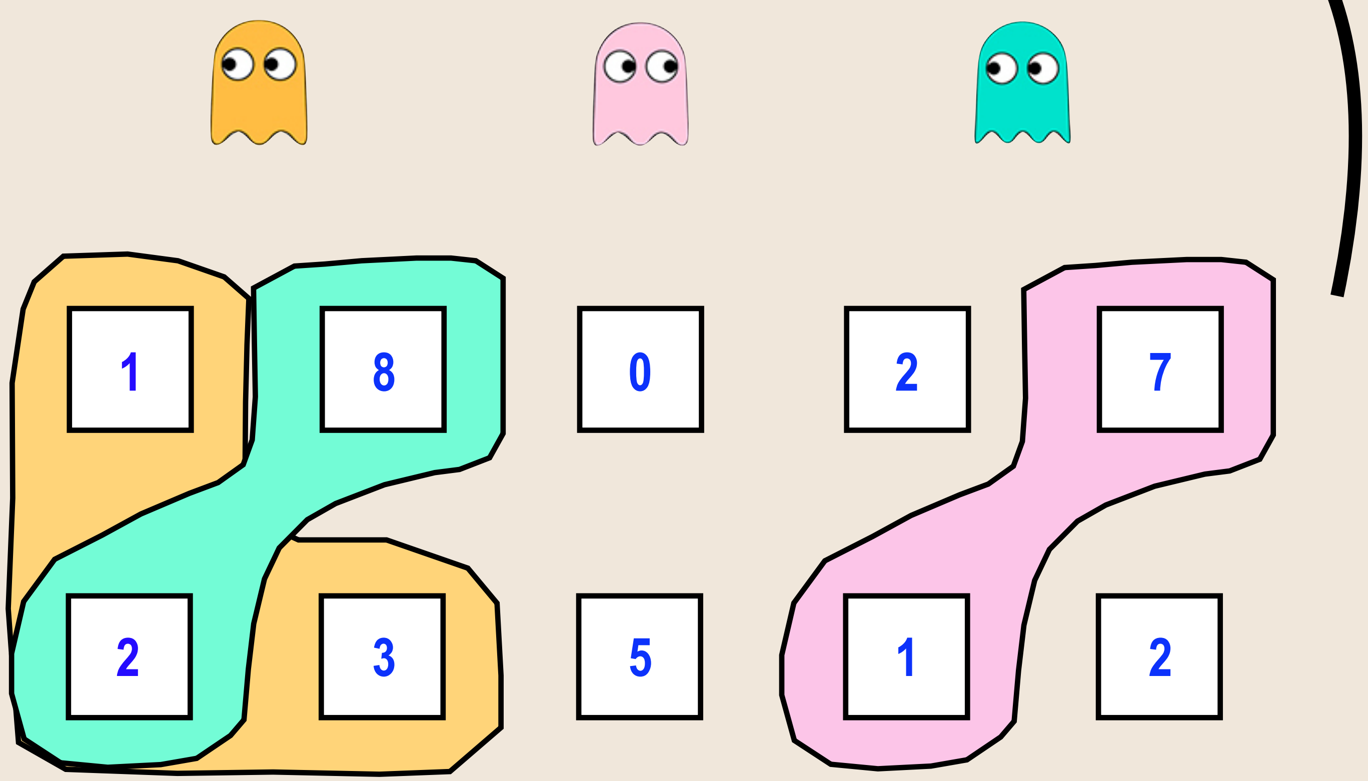
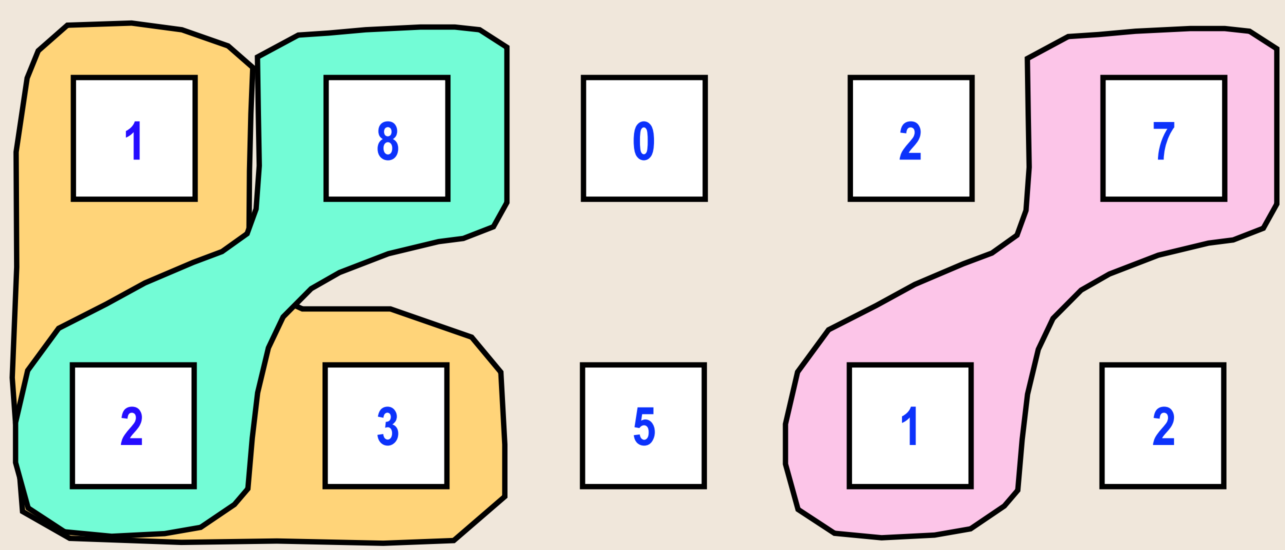
- Global Welfare: $W(x) = \sum_{r \in \cup x_i} v_r \; = 22$



redundancy ***not*** double counted

Solutions: Equilibrium to “designed” utility functions



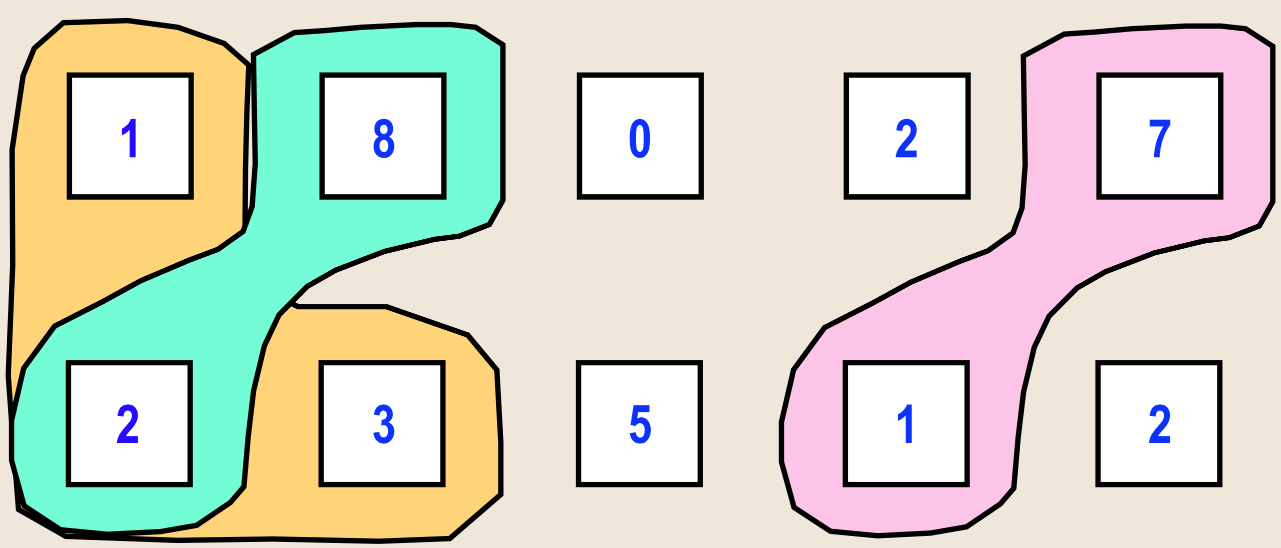
M. Gairing, “Covering Games: Approximations through Non-Cooperation”, 2009

Solutions: Equilibrium to "designed" utility functions

Utility functions:

- Structure: $U_i(x_i, x_{-i}) = \sum_{r \in x_i} v_r \cdot f(|x|_r)$
- Division rule: $f : \{0, 1, \ldots, n\} \to R$

$|x|_r =$ number agents choose r in allocation a



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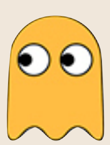
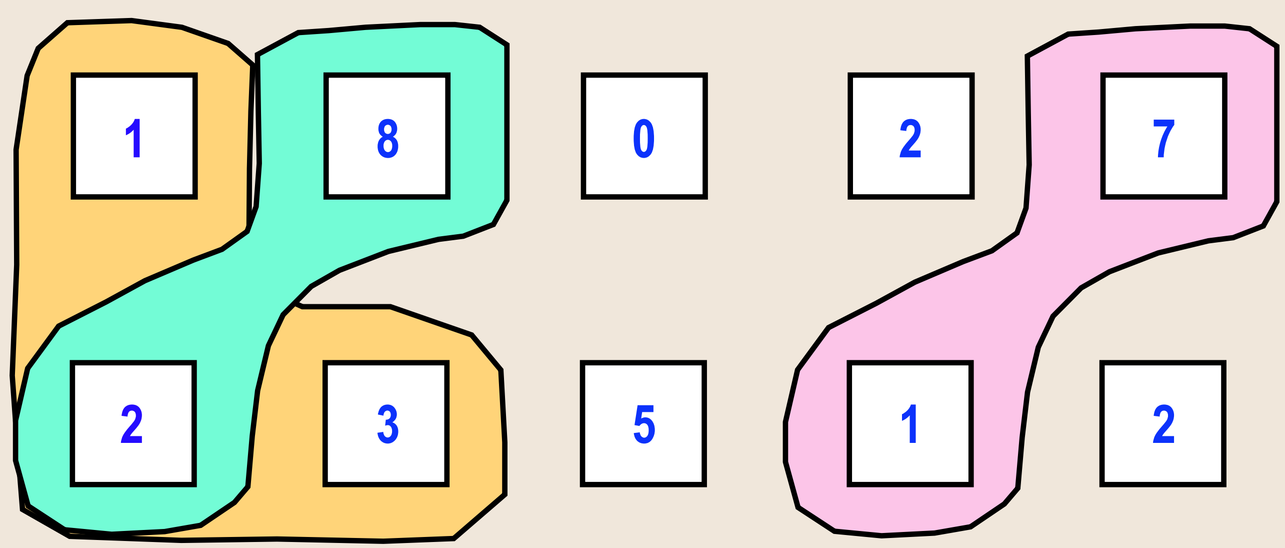


$$U = 1 \cdot f(1) + 2 \cdot f(2) + 3 \cdot f(1)$$



M. Gairing, "Covering Games: Approximations through Non-Cooperation", 2009

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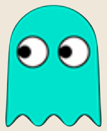
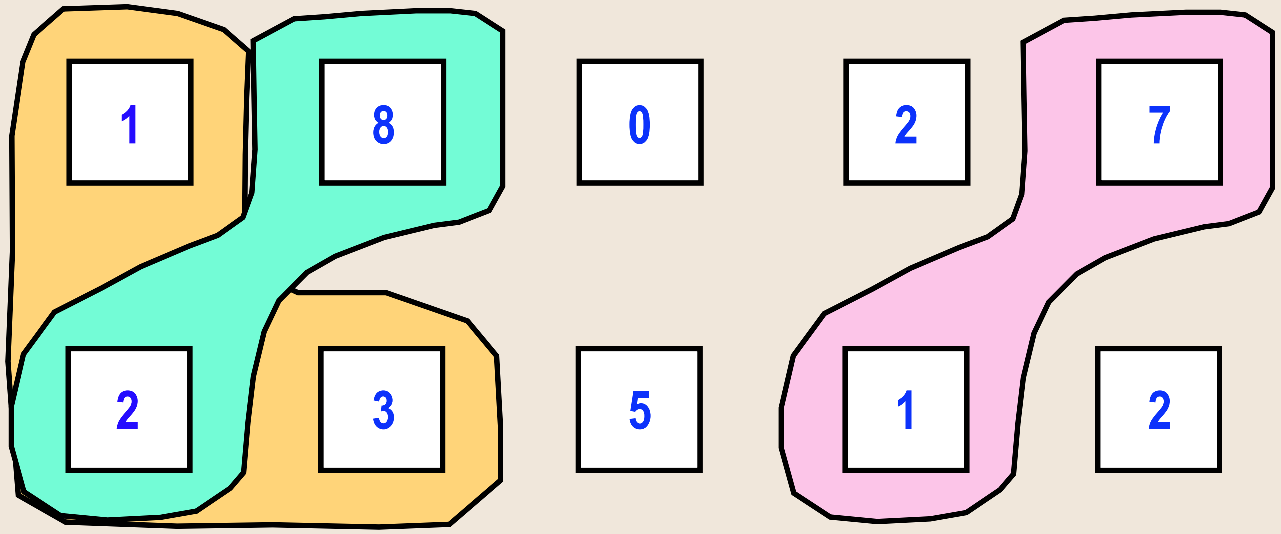


$$U = 2 \cdot f(2) + 8 \cdot f(1)$$



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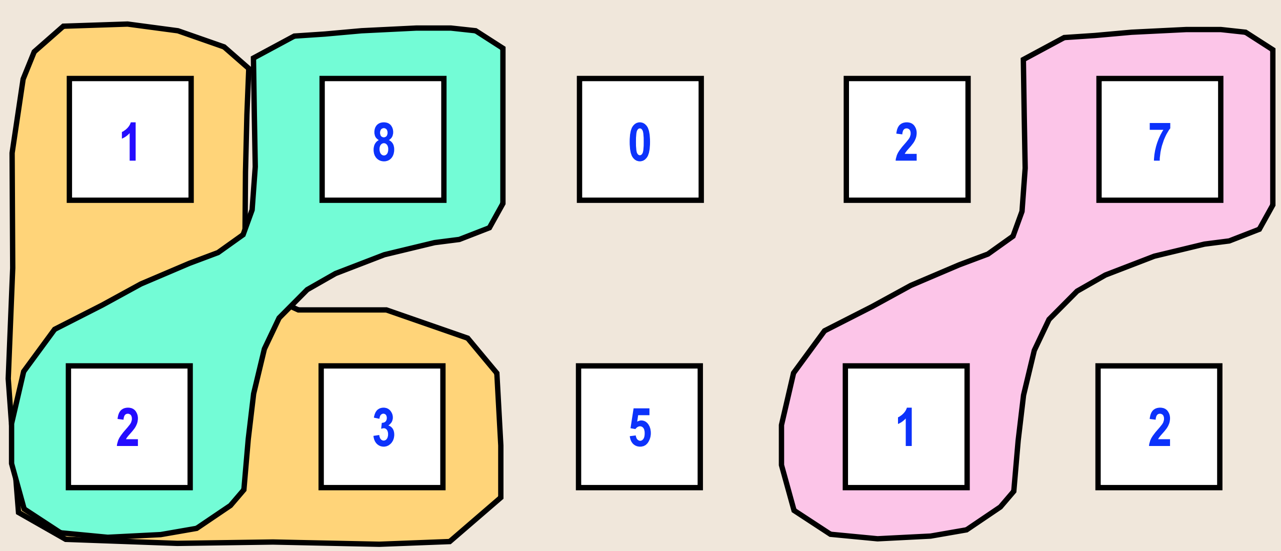
"optimal" design?



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game G

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game

G

set of games

$\mathcal{G}_f$

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game
G

set of games
$\mathcal{G}_f$

Solution concept:

- Nash equilibrium: $U_i(x^{\text{ne}}) \geq U_i(x_i, x^{\text{ne}}_{-i}), \; \forall i \in N, \; x_i \in X_i$

Setup:

- Resources: $\mathcal{R}$
- Values: $v_r \geq 0$
- Actions: $X_i \subseteq 2^{\mathcal{R}}, \; i \in N$
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game

G

↓

set of games

$\mathcal{G}_f$

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Goal: Design division rule f to optimize efficiency of resulting Nash equilibria

Model: Set covering

Setup:

- Resources: $\mathcal{R}$
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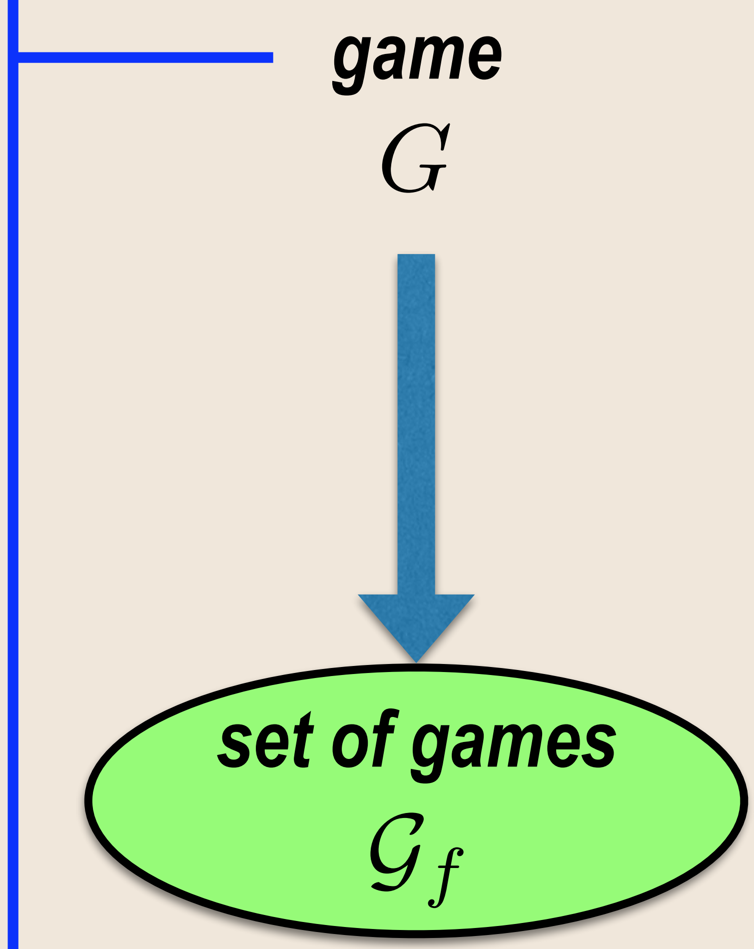

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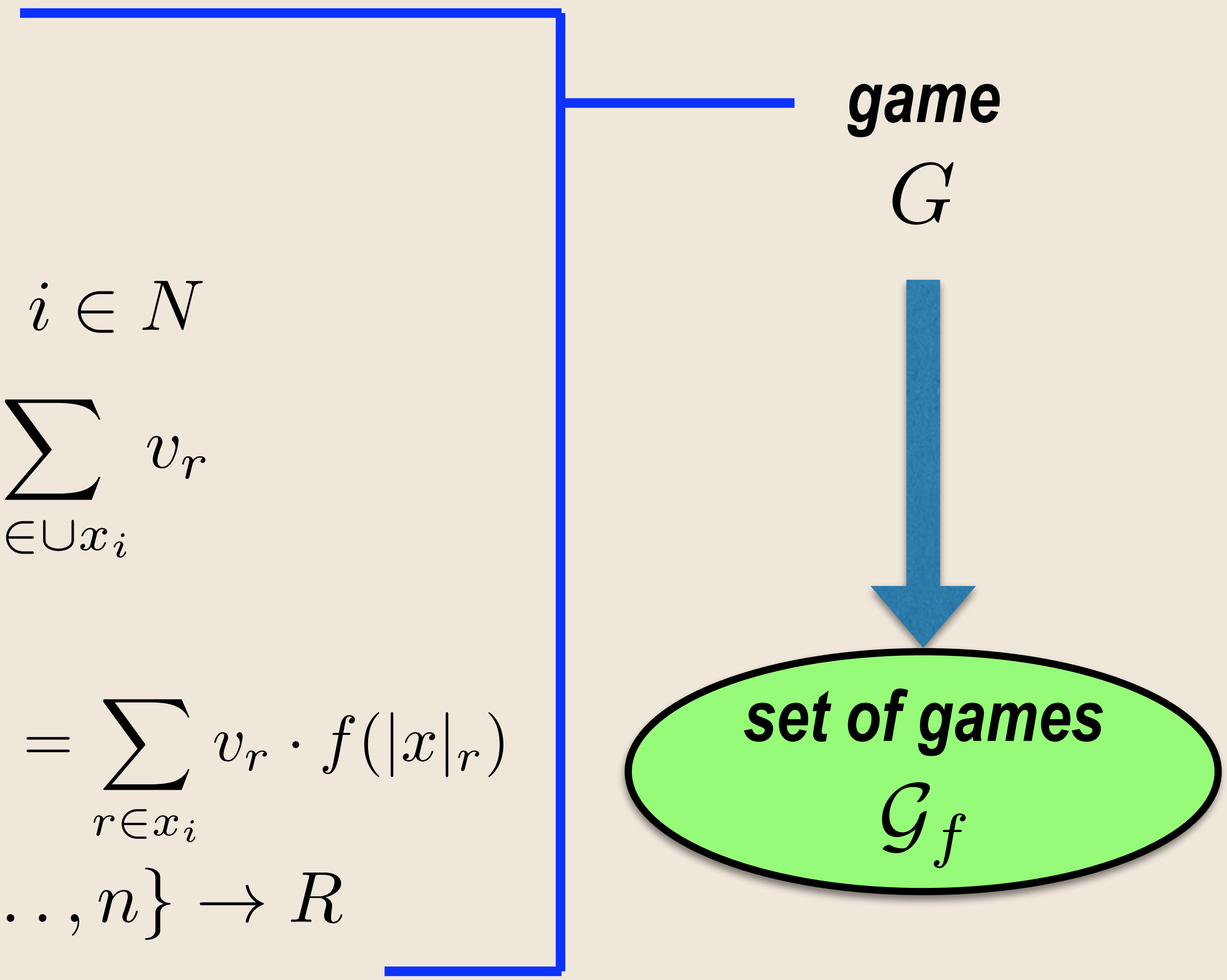

Performance measures:

- Price of anarchy (pessimistic) $\mathrm{PoA}(\mathcal{G}_f) = \min_{G \in \mathcal{G}_f} \left\{ \min_{x^{\mathrm{ne}} \in G} \left\{ \frac{W(x^{\mathrm{ne}})}{W(x^{\mathrm{opt}})} \right\} \right\}$

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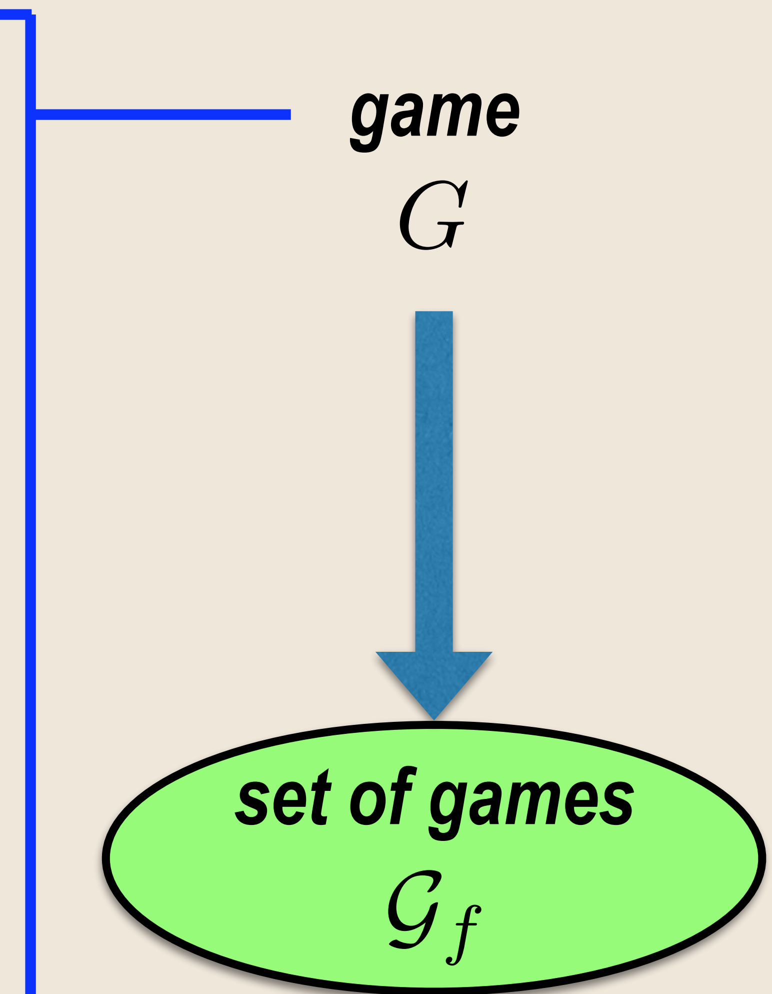

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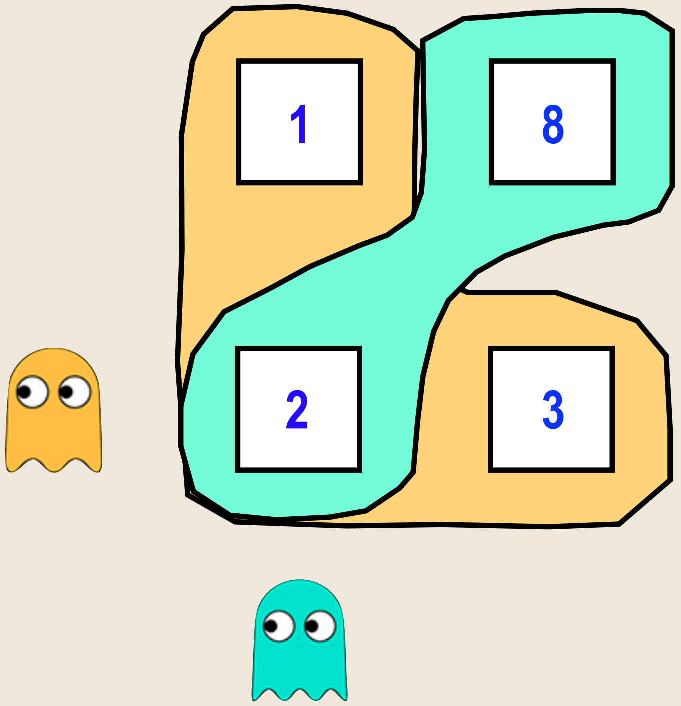
- Price of stability (optimistic): $\text{PoS}(\mathcal{G}_f) = \min_{G \in \mathcal{G}_f} \left\{ \max_{x^{\text{ne}} \in G} \left\{ \frac{W(x^{\text{ne}})}{W(x^{\text{opt}})} \right\} \right\}$

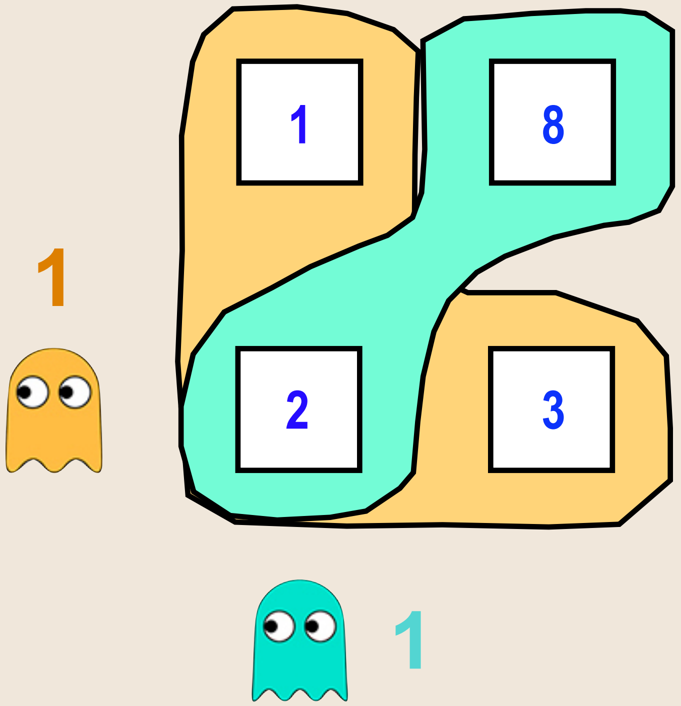
Goal: Design division rule f to optimize efficiency of resulting Nash equilibria



Goal: Design division rule f to optimize efficiency of resulting Nash equilibria

Design #1: Equal share (Shapley value)

$$f(k) = \frac{1}{k}$$



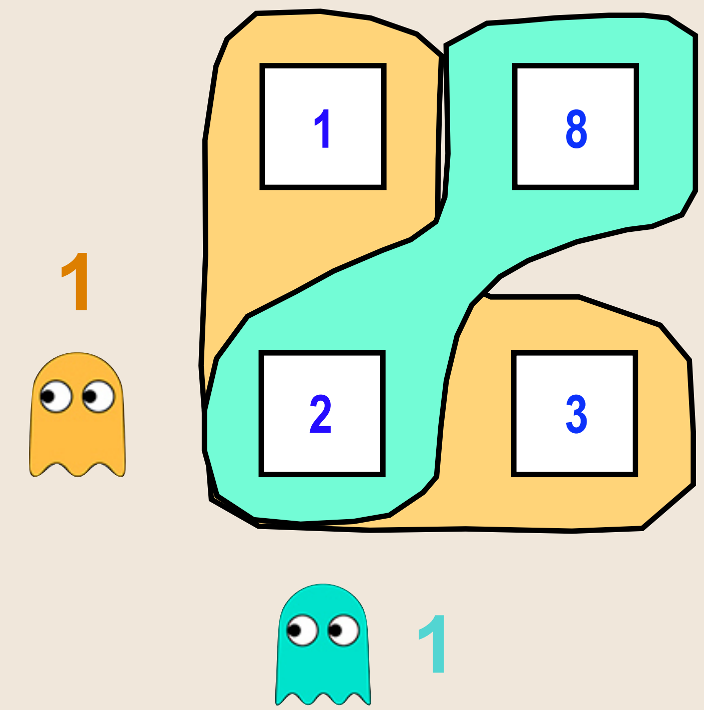
f(1) = 1
f(2) = 1/2
f(3) = 1/3

Design methodologies

Goal: Design division rule f to optimize efficiency of resulting Nash equilibria

Design #1: Equal share (Shapley value)

$$f(k) = \frac{1}{k} \quad \Rightarrow \quad \mathrm{PoA}(\mathcal{G}_f) = 1/2$$



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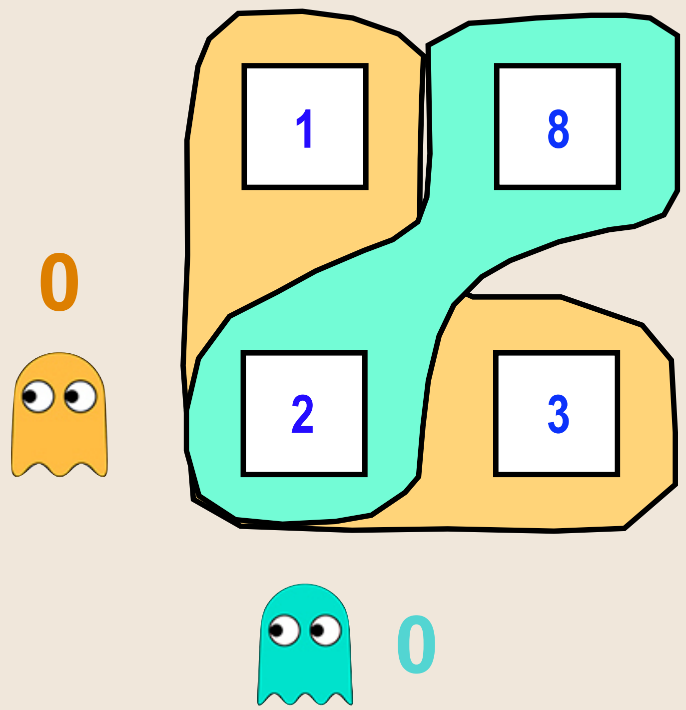

Design #2: Marginal contribution

$$f(1) = 1$$

$$f(2) = \cdots = f(n) = 0$$

f(1) = 1
f(2) = 0
f(3) = 0

Design methodologies

Goal: Design division rule f to optimize efficiency of resulting Nash equilibria

Design #1: Equal share (Shapley value)

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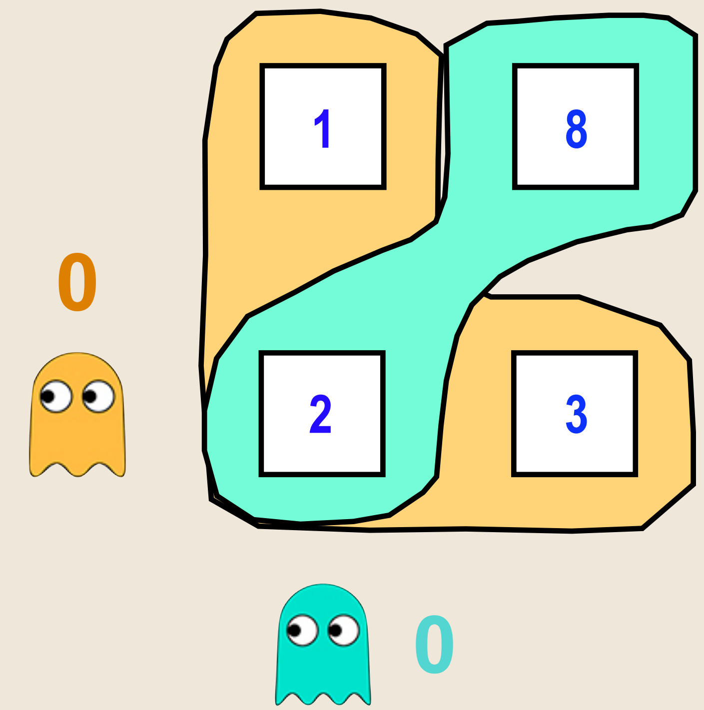

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Design methodologies

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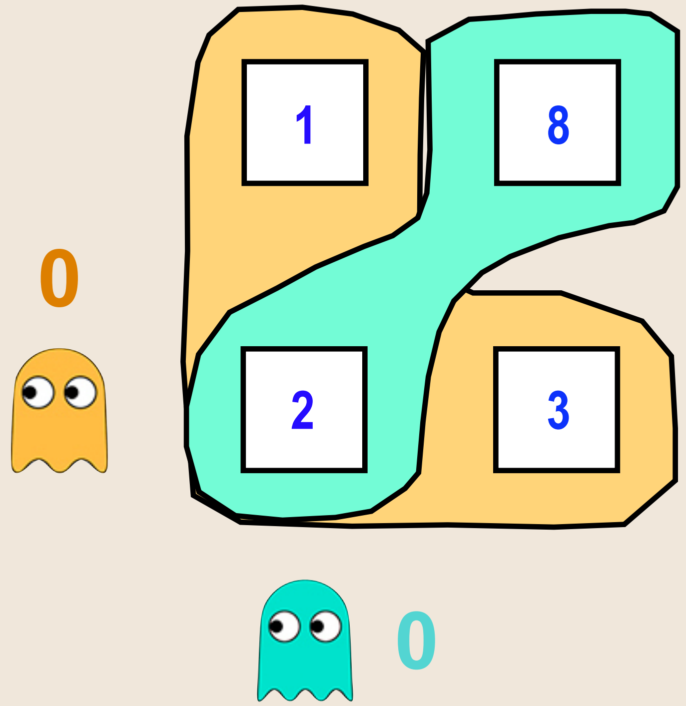

f(1) = 1
f(2) = 0
f(3) = 0

Equivalent to setting $U_i(x) = W(x)$

Goal: Design division rule f to optimize efficiency of resulting Nash equilibria

Design #1: Equal share (Shapley value)

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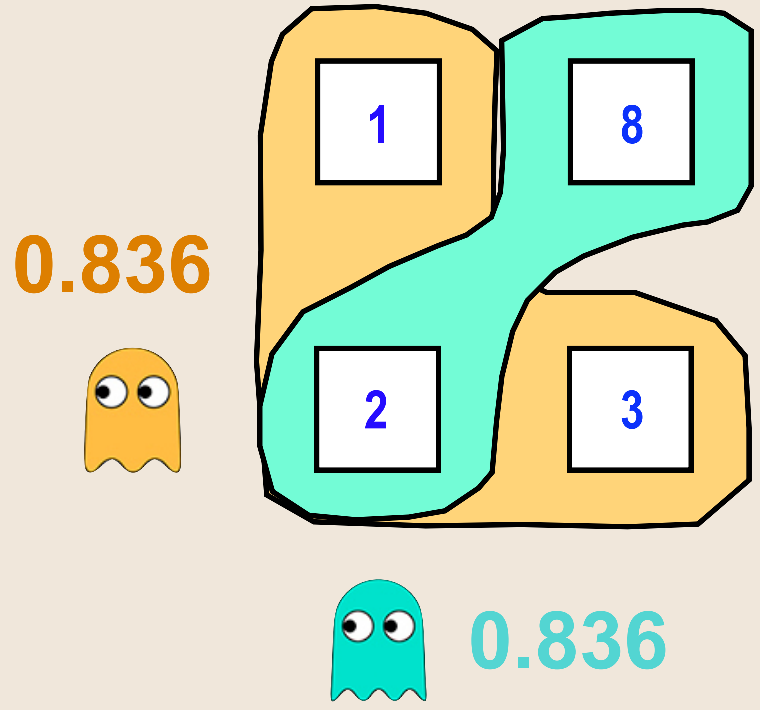


Design #2: Marginal contribution

$$f(1) = 1 \quad \Rightarrow \quad \mathrm{PoA}(\mathcal{G}_f) = 1/2$$

$$f(2) = \cdots = f(n) = 0$$

f(1) = 1
f(2) = 0.418
f(3) = 0.254

Design #3: Gairing's rule

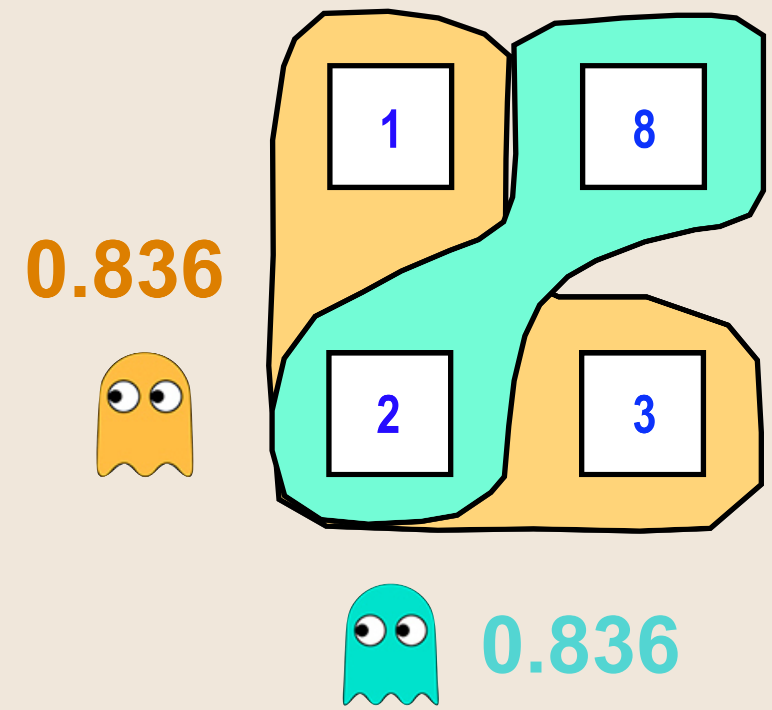
$$f(k) = (k-1)! \left(\frac{\sum_{i=k}^{\infty} \frac{1}{i!}}{\sum_{i=1}^{\infty} \frac{1}{i!}} \right)$$

M. Gairing, "Covering Games: Approximations through Non-Cooperation", 2009

Goal: Design division rule f to optimize efficiency of resulting Nash equilibria

Design #1: Equal share (Shapley value)

$$f(k) = \frac{1}{k} \quad \longrightarrow \quad \text{PoA}(\mathcal{G}_f) = 1/2$$



Design #2: Marginal contribution

$$f(1) = 1 \quad \longrightarrow \quad \text{PoA}(\mathcal{G}_f) = 1/2$$

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Design #3: Gairing's rule

$$f(k) = (k-1)! \left(\frac{\sum_{i=k}^{\infty} \frac{1}{i!}}{\sum_{i=1}^{\infty} \frac{1}{i!}} \right) \quad \longrightarrow \quad \text{PoA}(\mathcal{G}_f) = 1 - 1/e \approx 0.63$$

M. Gairing, "Covering Games: Approximations through Non-Cooperation", 2009

Goal: Design division rule f to optimize efficiency of resulting Nash equilibria

Design #1: Equal share (Shapley value)

$$f(k) = \frac{1}{k} \quad \Rightarrow \quad \text{PoA}(\mathcal{G}_f) = 1/2$$

Design #2: Marginal contribution

$$f(1) = 1 \quad \Rightarrow \quad \text{PoA}(\mathcal{G}_f) = 1/2$$

$$f(2) = \cdots = f(n) = 0$$

Design #3: Gairing's rule **(optimizes price of anarchy)**

$$f(k) = (k-1)! \left(\frac{\sum_{i=k}^{\infty} \frac{1}{i!}}{\sum_{i=1}^{\infty} \frac{1}{i!}} \right) \quad \Rightarrow \quad \text{PoA}(\mathcal{G}_f) = 1 - 1/e \approx 0.63$$

M. Gairing, "Covering Games: Approximations through Non-Cooperation", 2009

Goal: Design division rule f to optimize efficiency of resulting Nash equilibria

{ price of anarchy? / price of stability? }

Design #1: Equal share (Shapley value)

$$f(k) = \frac{1}{k} \;\Longrightarrow\; \mathrm{PoA}(\mathcal{G}_f) = 1/2$$

Design #2: Marginal contribution

$$f(1) = 1 \;\Longrightarrow\; \mathrm{PoA}(\mathcal{G}_f) = 1/2$$

$$f(2) = \cdots = f(n) = 0$$

Design #3: Gairing's rule **(optimizes price of anarchy)**

$$f(k) = (k-1)!\left(\frac{\sum_{i=k}^{\infty} \frac{1}{i!}}{\sum_{i=1}^{\infty} \frac{1}{i!}}\right) \;\Longrightarrow\; \mathrm{PoA}(\mathcal{G}_f) = 1 - 1/e \approx 0.63$$

M. Gairing, "Covering Games: Approximations through Non-Cooperation", 2009

Goal: Design division rule f to optimize efficiency of resulting Nash equilibria

- price of anarchy?
- price of stability?

Design #1: Equal share (Shapley value)

$$f(k) = \frac{1}{k} \Rightarrow \mathrm{PoA}(\mathcal{G}_f) = 1/2, \quad \mathrm{PoS}(\mathcal{G}_f) = 1/2$$

Design #2: Marginal contribution

$$f(1) = 1, \quad f(2) = \cdots = f(n) = 0 \Rightarrow \mathrm{PoA}(\mathcal{G}_f) = 1/2$$

Design #3: Gairing's rule

$$f(k) = (k-1)!\left(\frac{\sum_{i=k}^{\infty} \frac{1}{i!}}{\sum_{i=1}^{\infty} \frac{1}{i!}}\right) \Rightarrow \mathrm{PoA}(\mathcal{G}_f) = 1 - 1/e \approx 0.63$$

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$$f(1) = 1, \quad f(2) = \cdots = f(n) = 0 \Rightarrow \mathrm{PoA}(\mathcal{G}_f) = 1/2, \quad \mathrm{PoS}(\mathcal{G}_f) = 1$$

Design #3: Gairing's rule

$$f(k) = (k-1)!\left(\frac{\sum_{i=k}^{\infty}\frac{1}{i!}}{\sum_{i=1}^{\infty}\frac{1}{i!}}\right) \Rightarrow \mathrm{PoA}(\mathcal{G}_f) = 1 - 1/e \approx 0.63$$

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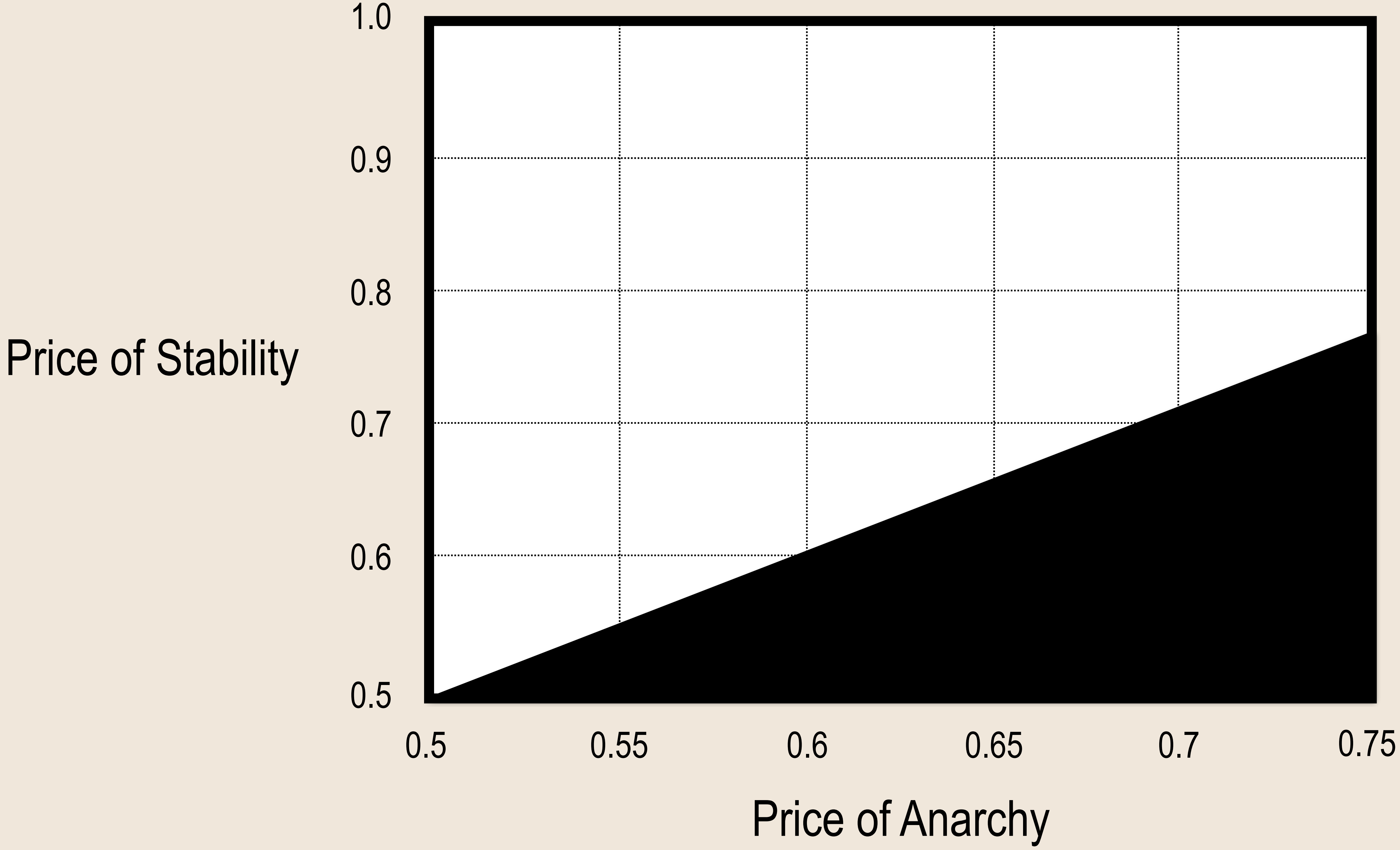
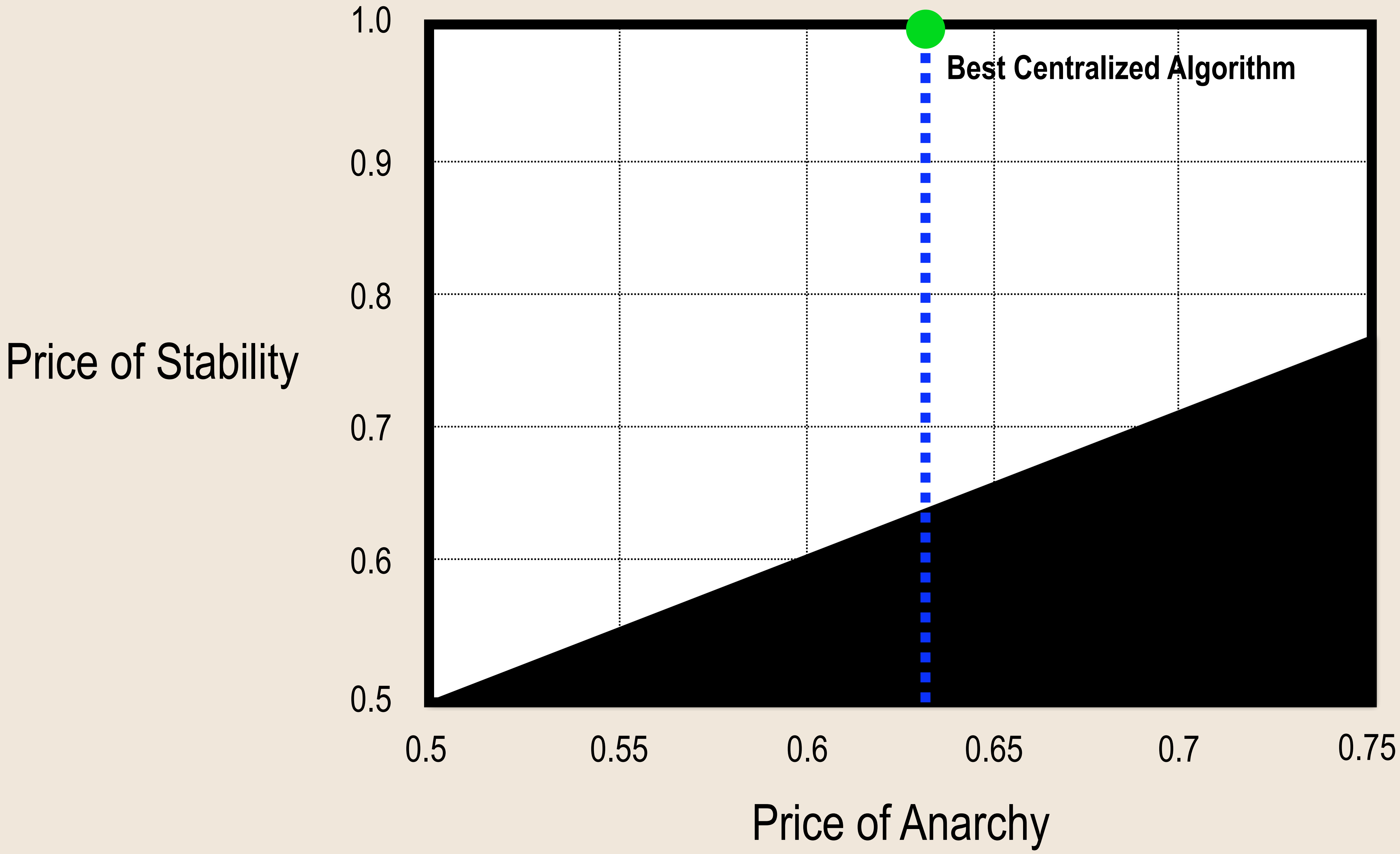
Tradeoff?

Design #3: Gairing's rule

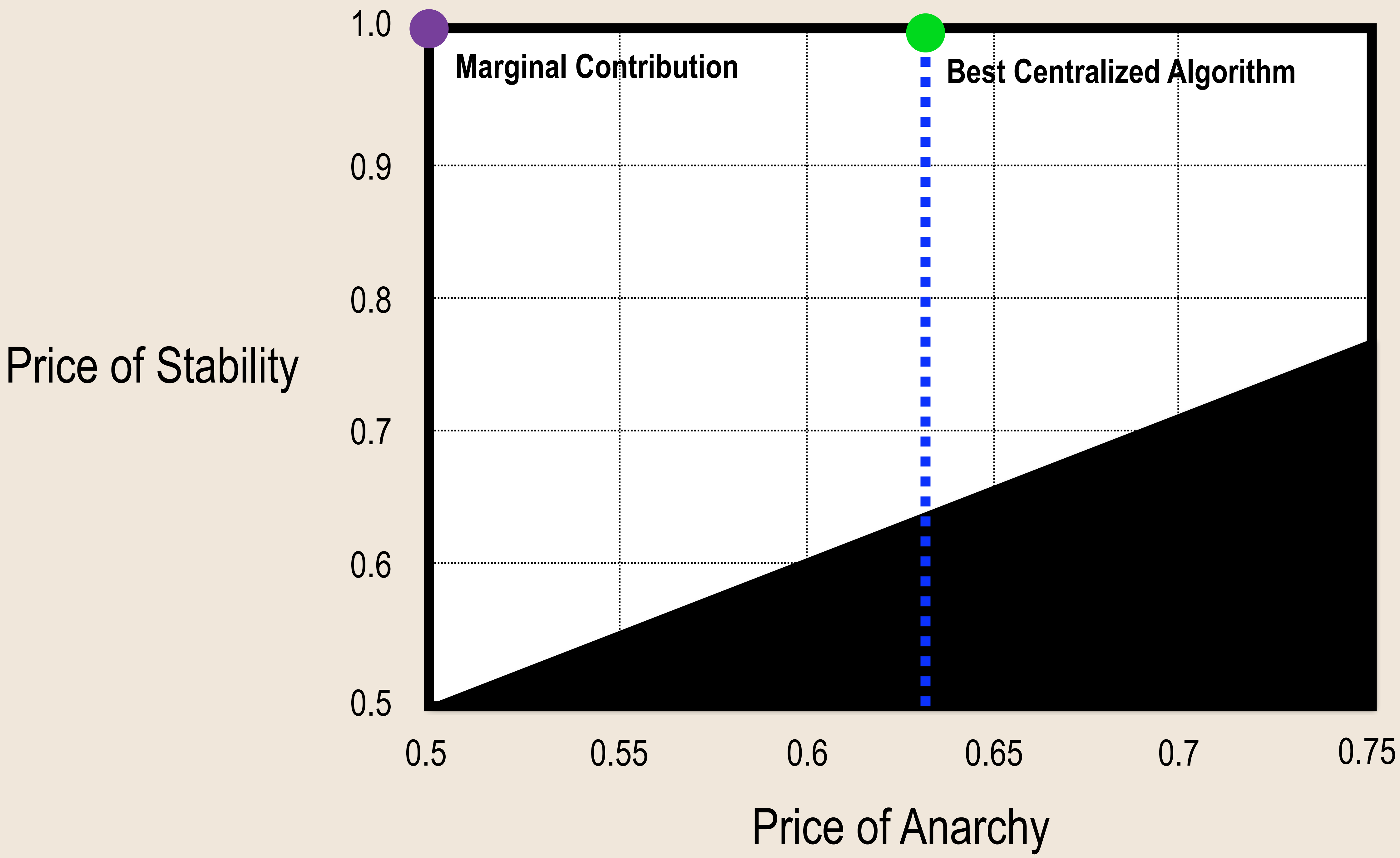
$$f(k) = (k-1)!\left(\frac{\sum_{i=k}^{\infty}\frac{1}{i!}}{\sum_{i=1}^{\infty}\frac{1}{i!}}\right) \Rightarrow \mathrm{PoA}(\mathcal{G}_f) = 1 - 1/e \approx 0.63, \quad \mathrm{PoS}(\mathcal{G}_f) = 1 - 1/e$$


1.0
0.9
0.8
0.7
0.6
0.5
Price of Stability
0.5
0.55
0.6
0.65
0.7
0.75
Price of Anarchy

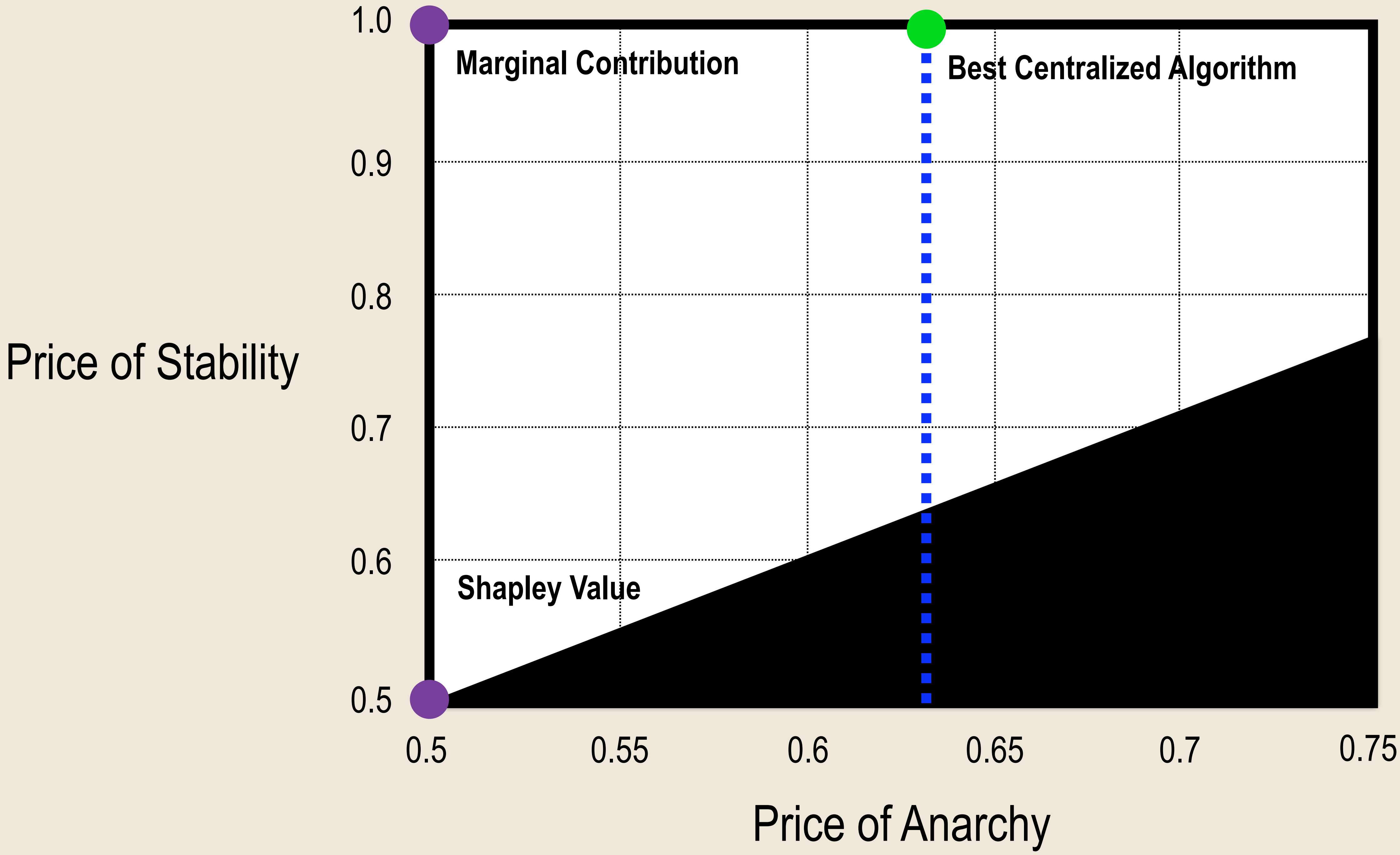
Achievable efficiencies



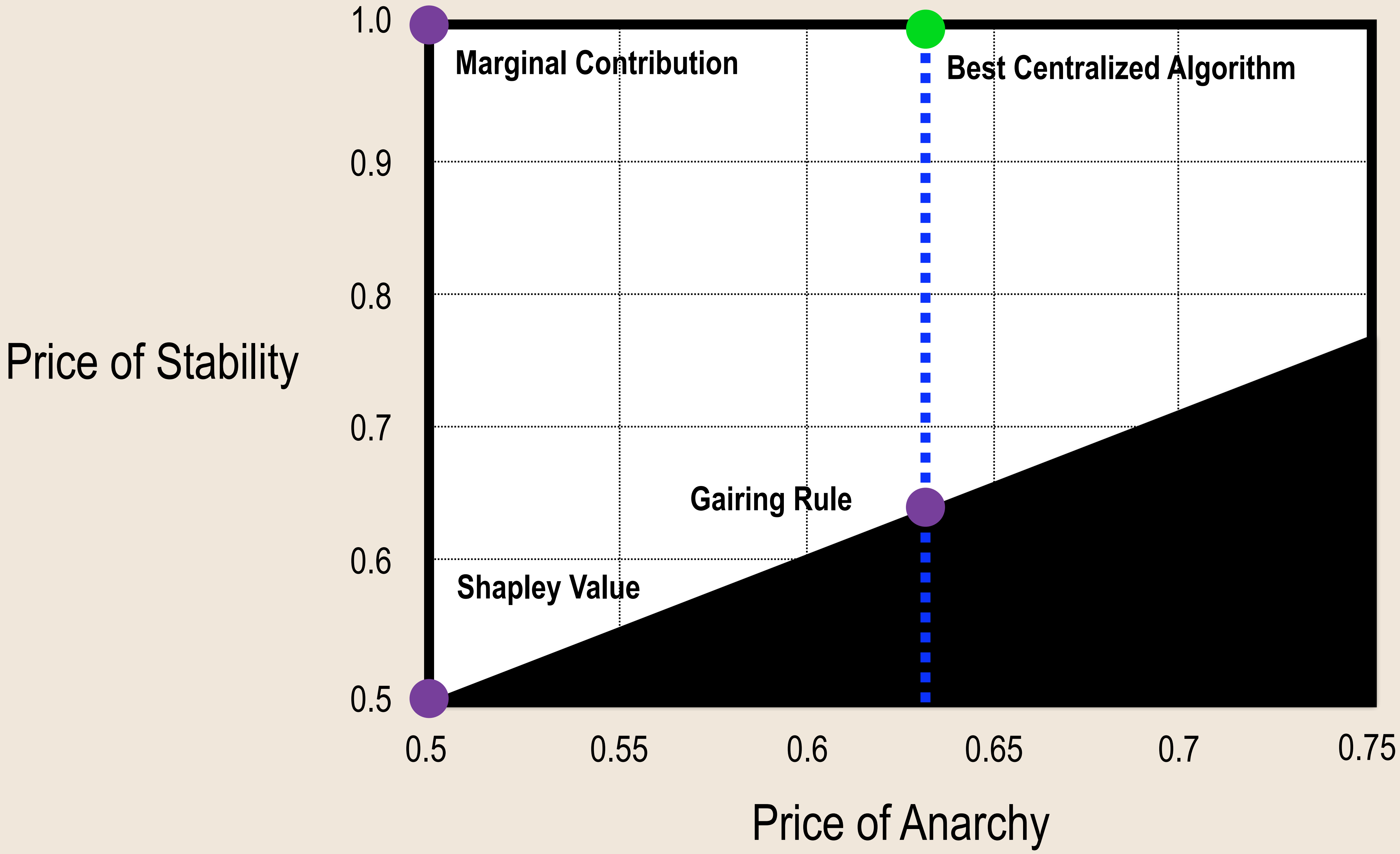
Achievable efficiencies



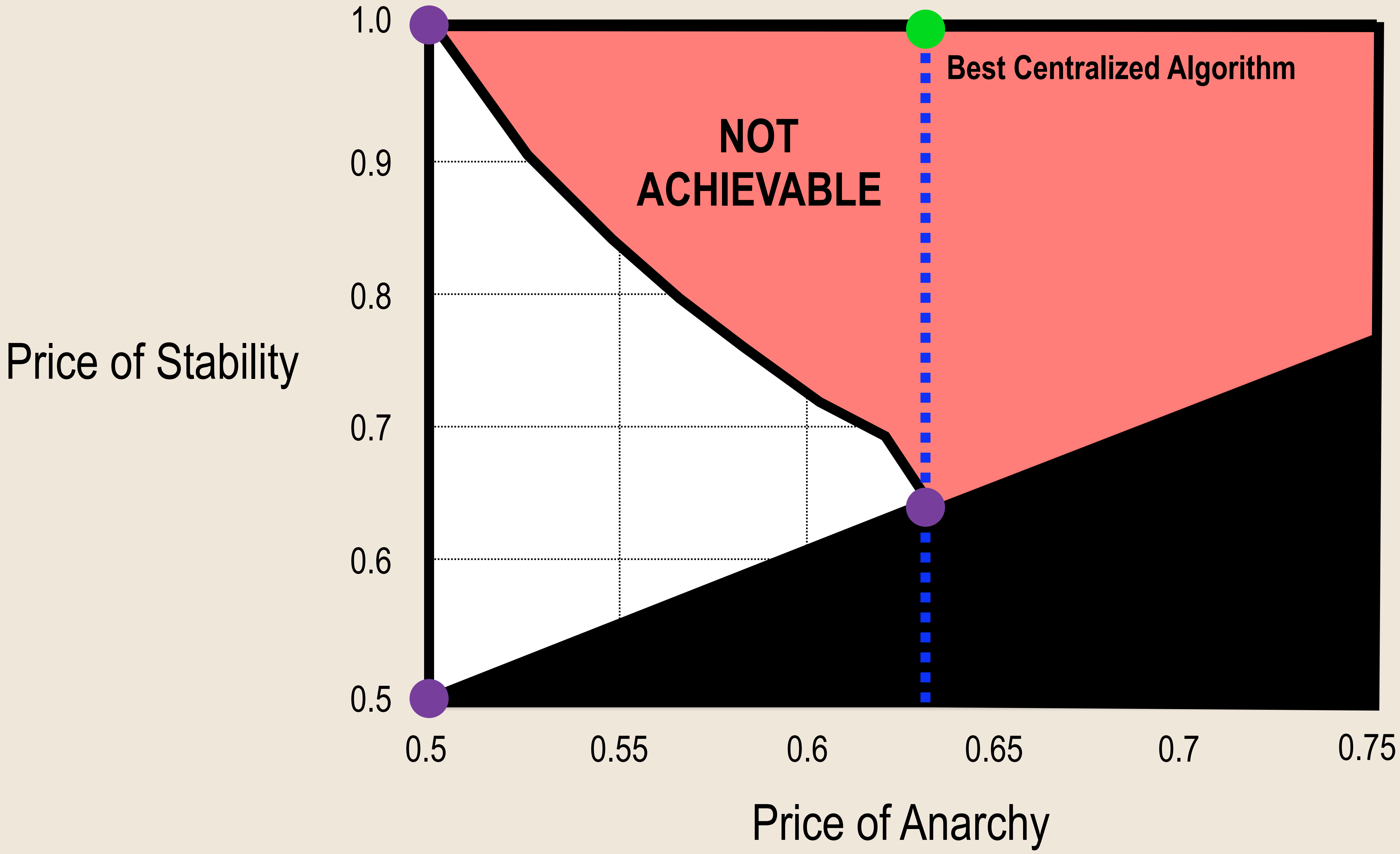
Achievable efficiencies

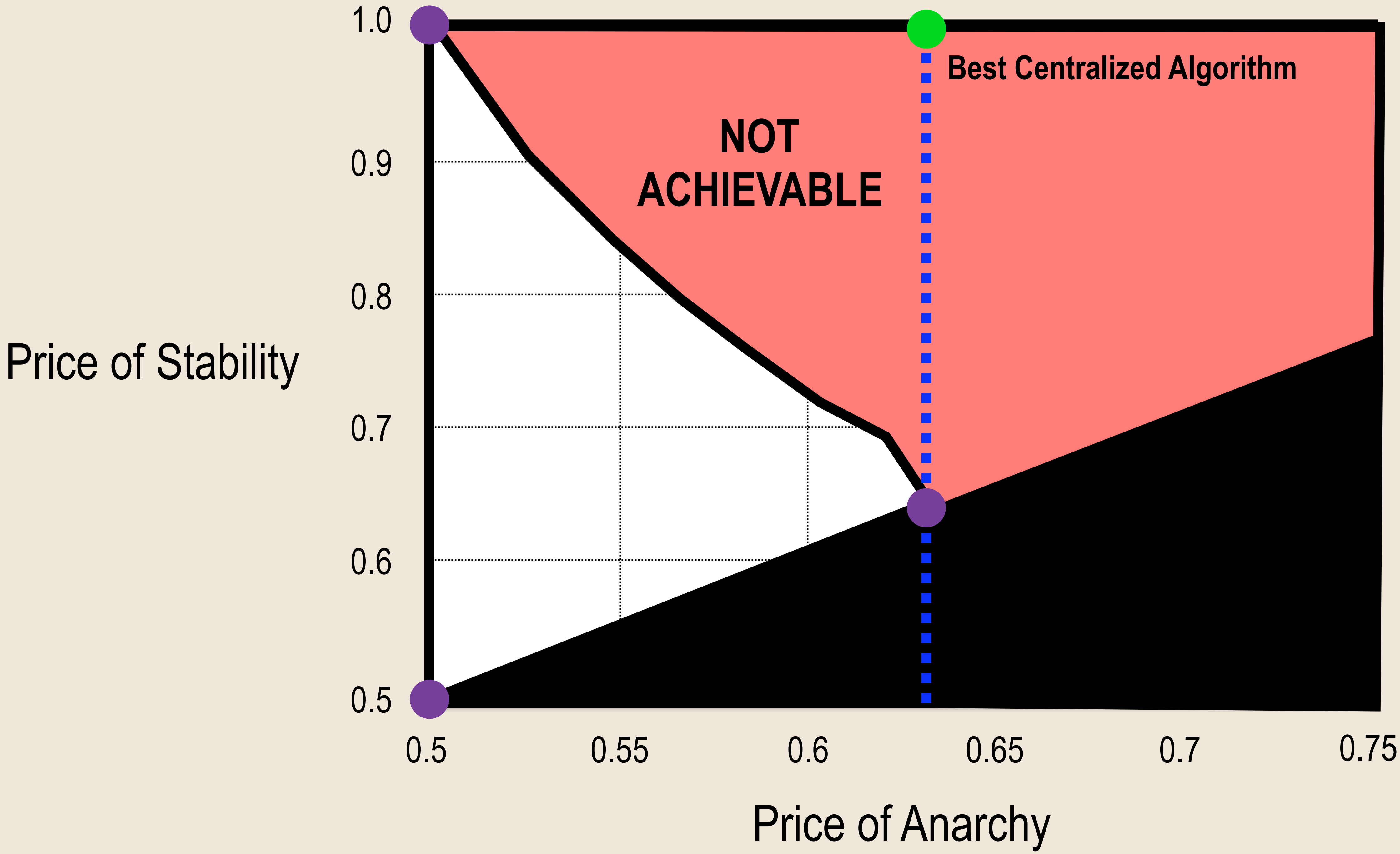


Achievable efficiencies

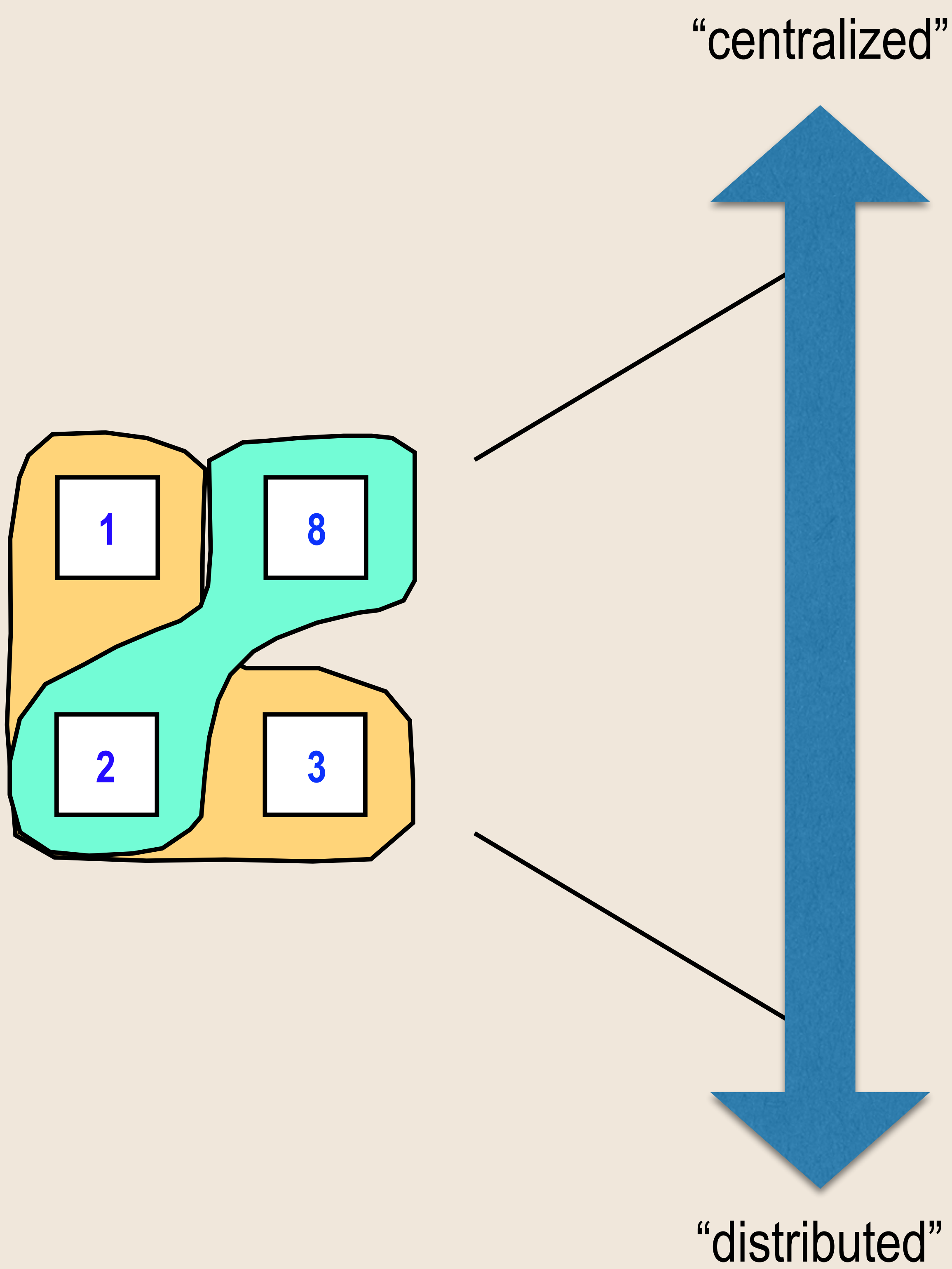


Achievable efficiencies





Main Result: Inherent tension between price of anarchy and price of stability



$$\text{PoS} = 1$$
$$\text{PoA} = 1 - 1/e$$

two main driving factors in inefficiency

decision-making architecture

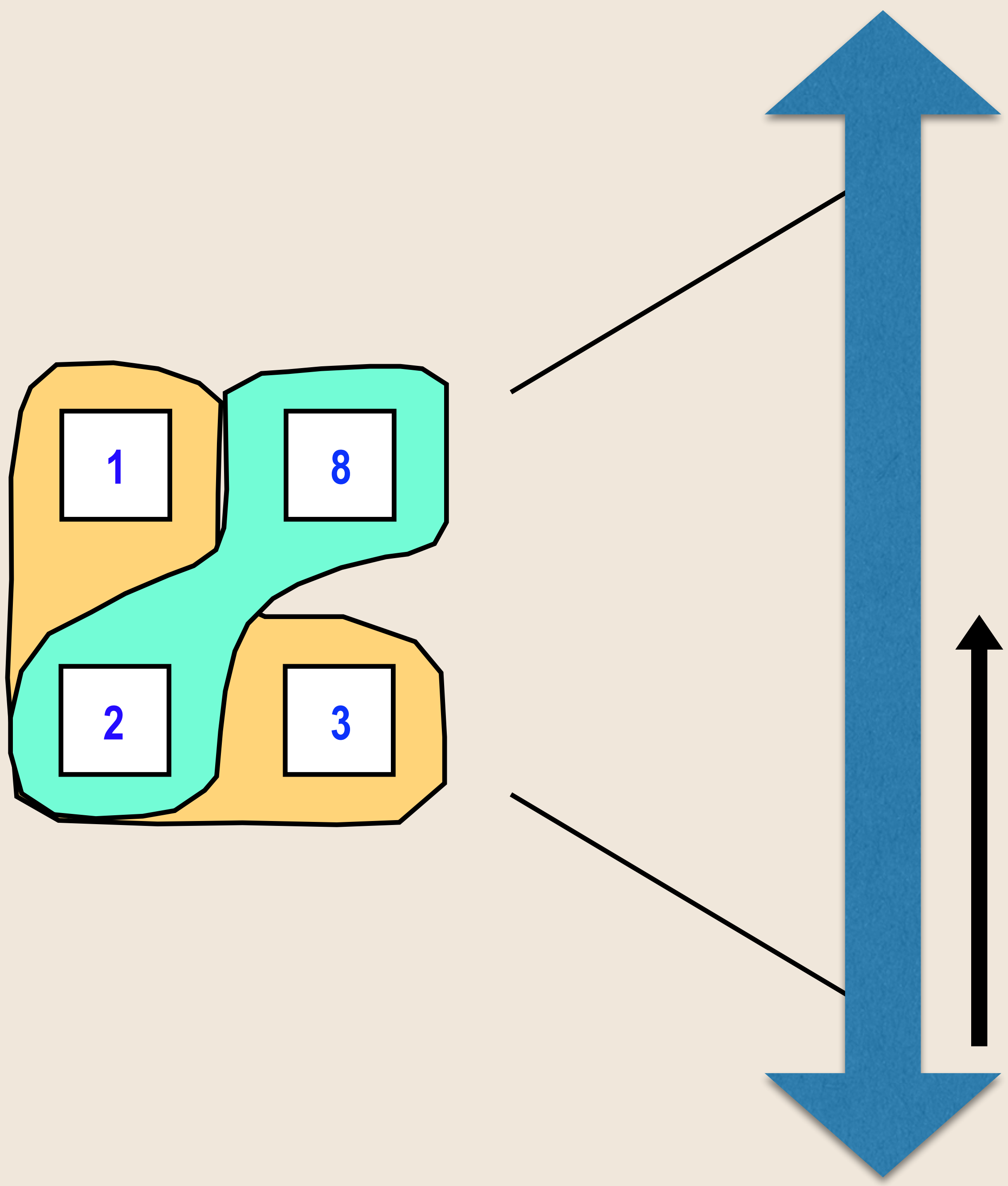
decision-making process

$$\text{PoS} = 1 - 1/e$$
$$\text{PoA} = 1 - 1/e$$

"centralized"

$$\mathrm{PoS} = 1$$

$$\mathrm{PoA} = 1 - 1/e$$



two main driving factors in inefficiency

decision-making architecture

decision-making process

improving performance guarantees
requires changing architecture

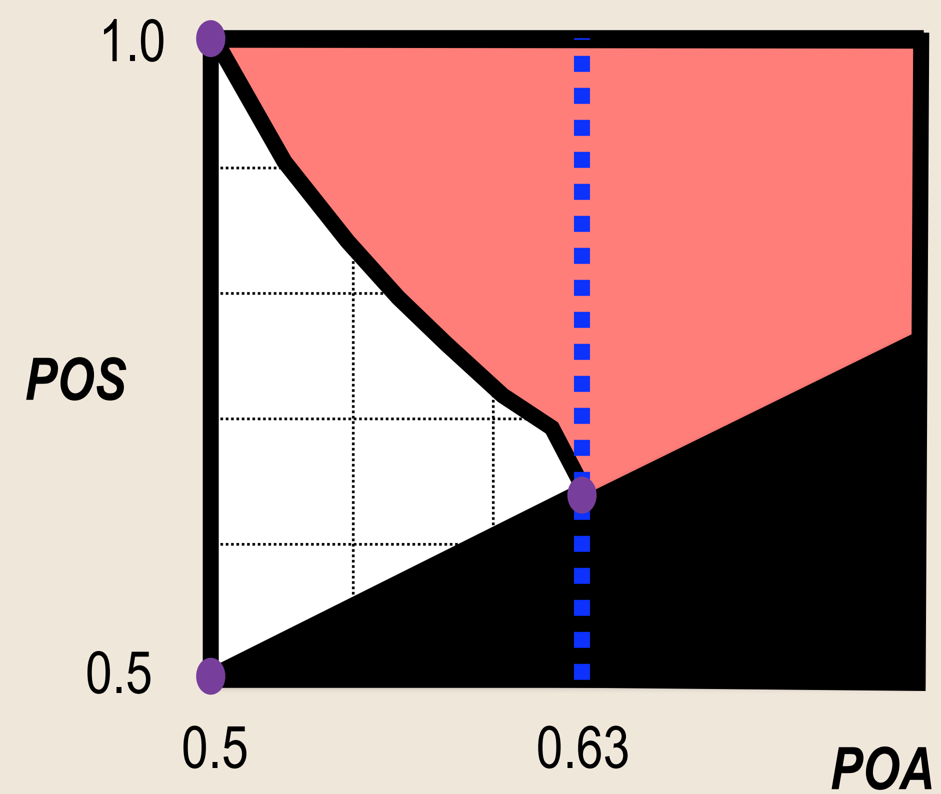
"distributed"

$$\mathrm{PoS} = 1 - 1/e$$

$$\mathrm{PoA} = 1 - 1/e$$

Setup:

- Resources: $\mathcal{R}$
- Values: $v_r \geq 0$
- Actions: $X_i \subseteq 2^{\mathcal{R}}, \; i \in N$
- Global Welfare: $W(x) = \sum_{r \in \cup x_i} v_r$



Design elements:

- Utility functions: $U_i(x_i, x_{-i}) = \sum_{r \in x_i} v_r \cdot f(|x|_r)$
- Division rule: $f : \{0, 1, \ldots, n\} \rightarrow R$

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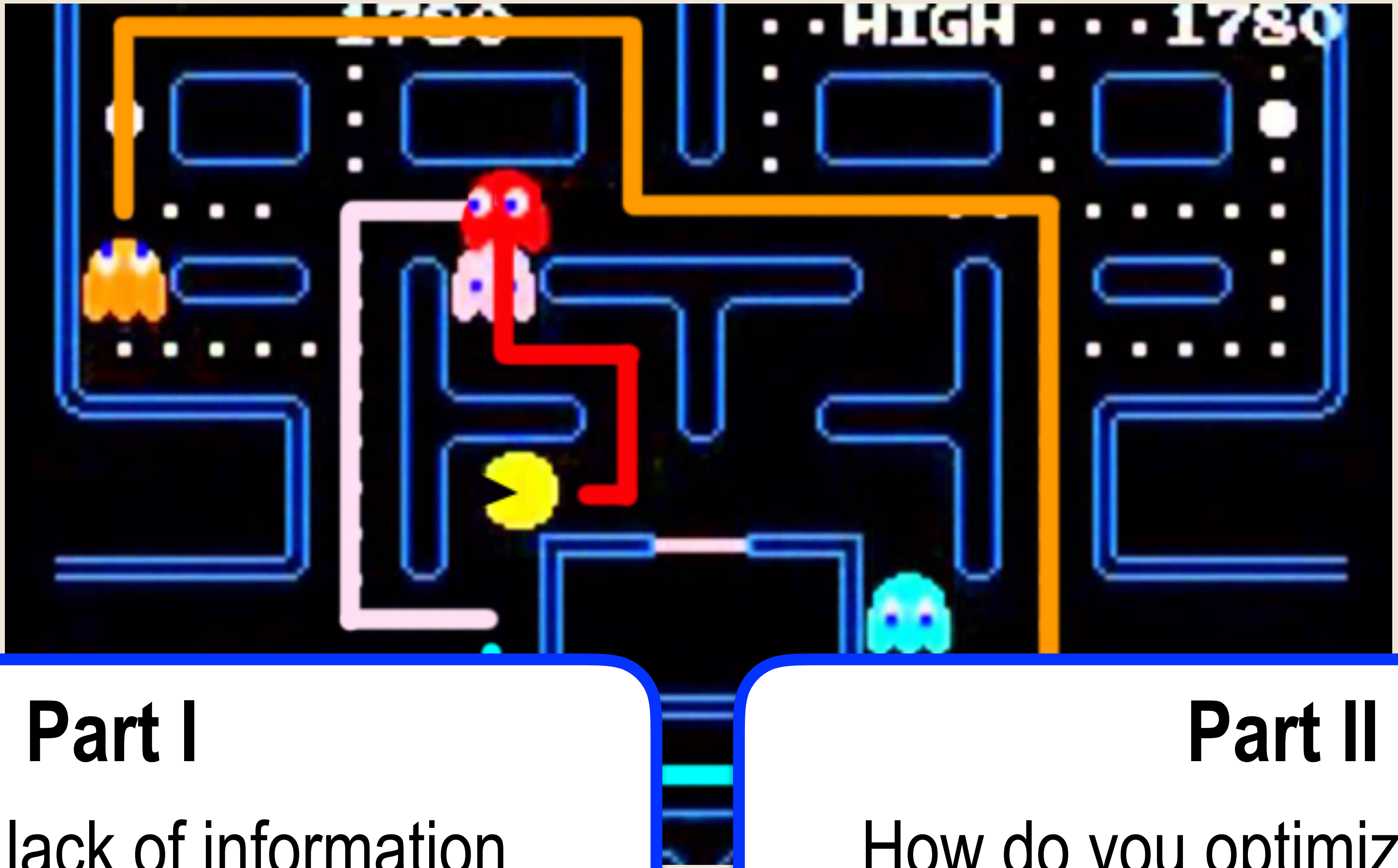
Main Results: [Paccagnan, Chandan, JRM, 2019]

- Systematic approach (LP) for characterizing POA for $\{(W_r, f_r) : r \in \mathcal{R}\}$
- Systematic approach (LP) for optimizing POA for $\{(W_r, f_r^{\text{opt}}) : r \in \mathcal{R}\}$

Central Goal

design of ***admissible*** control algorithms that attain ***near-optimal*** system-wide behavior in a ***reasonable*** period of time



Part I

How does lack of information degrade achievable performance?

Part II

How do you optimize collective performance using information?

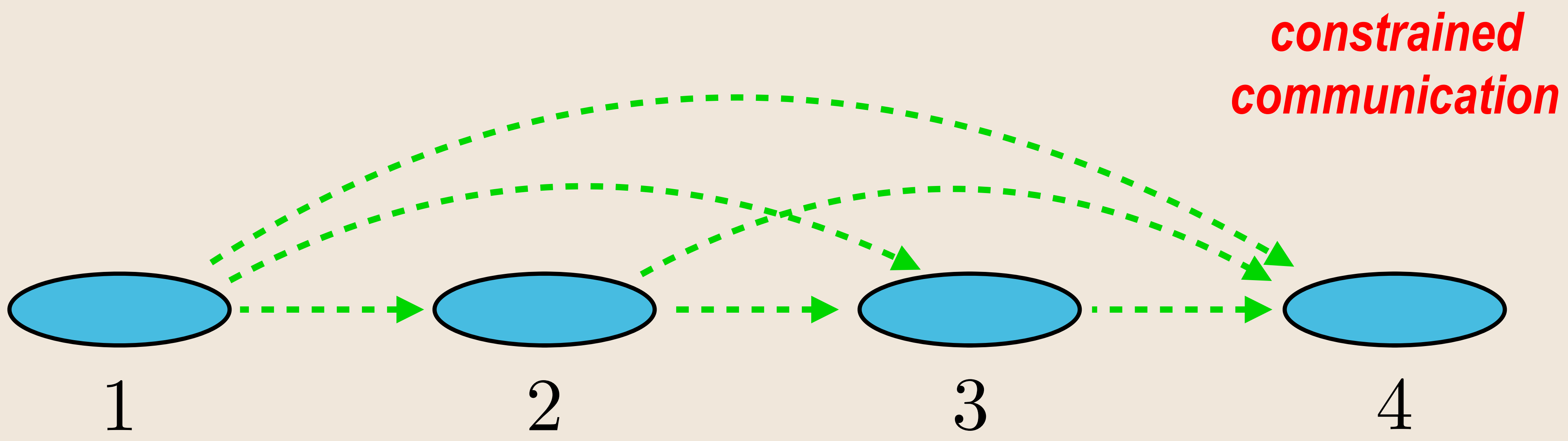
Thank you

Highlighted papers:

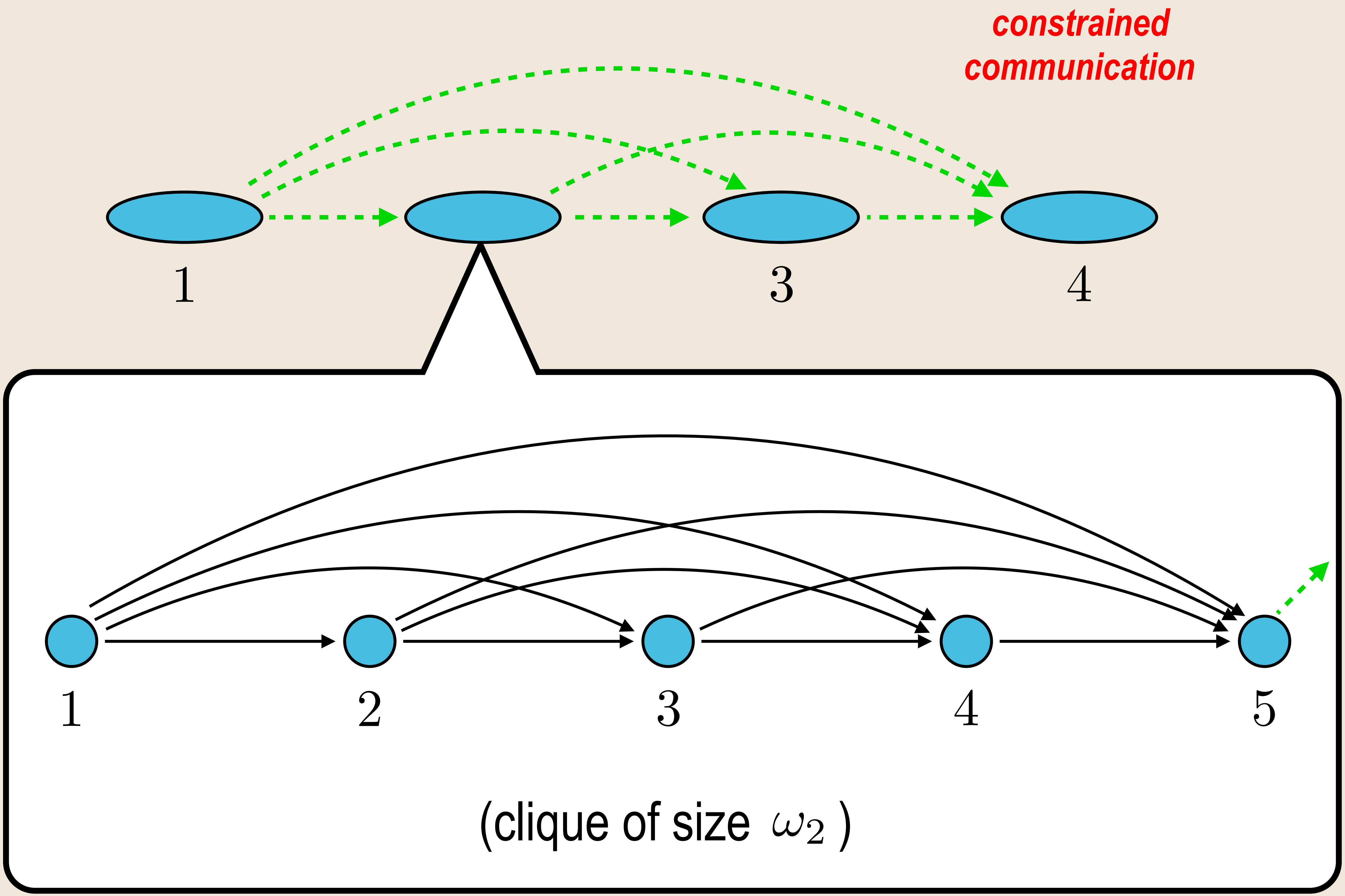
- D. Grimsman et al., “Value of Information in Greedy Submodular Maximization,” TCNS, 2019.
- D. Grimsman et al., “Strategic Information Sharing in Greedy Submodular Maximization,” CDC, 2018.
- V. Ramaswamy et al., “Multiagent Coverage Problems: The Trade-offs Between Anarchy and Stability,” 2019 (in review).
- D. Paccagnan, R. Chandan, & JRM, “Distributed Resource Allocation Through Utility Design - Part I: Optimizing the Performance Certificates via the Price of Anarchy,” 2019 (in review)
- D. Paccagnan & JRM, “Distributed Resource Allocation Through Utility Design - Part II: Applications to Submodular, Supermodular, and Set Covering Problems,” 2019 (in review)

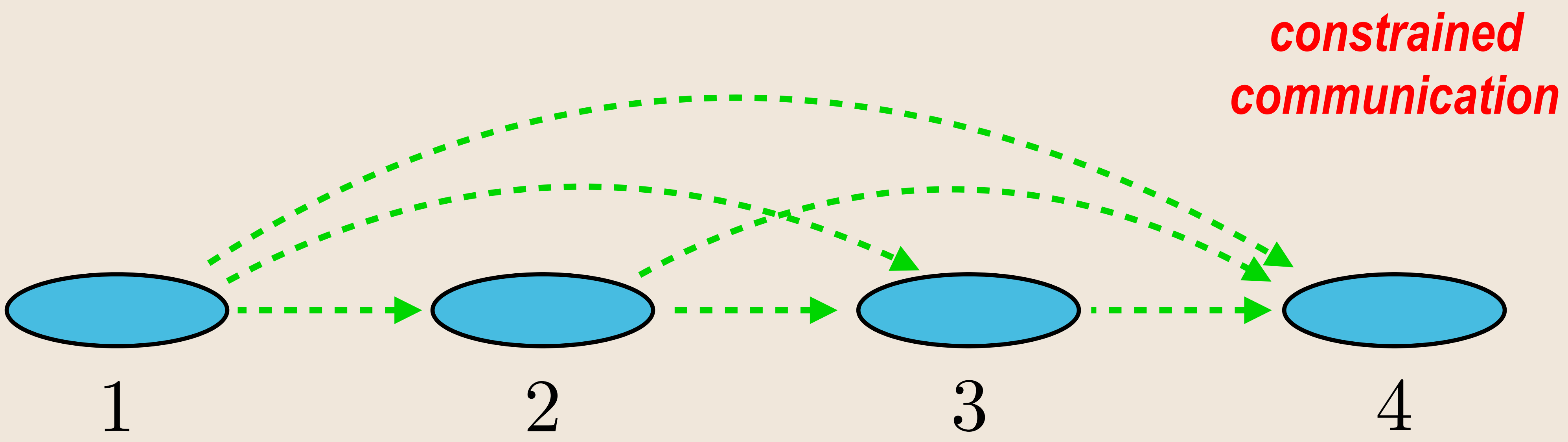
Relevant Papers:

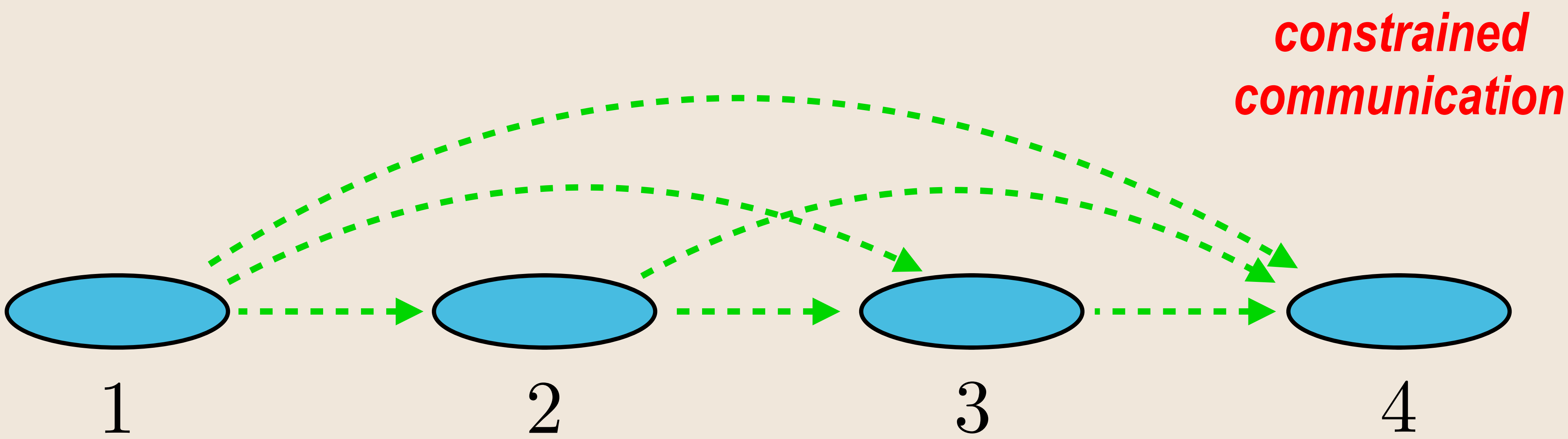
- B. Gharesifard and S. L. Smith, “Distributed submodular maximization with limited information,” TCNS, 2017.
- JRM, “The role of information in distributed resource allocation” TCNS, 2017.
- G. Qu et al., “Distributed greedy algorithm for mulit-agent task assignment problem with submodular utility functions,” 2017.
- B. Mirzasoleiman et al., “Distributed submodular maximization: Identifying representative elements in massive data,” 2013.



series of m disconnected cliques


constrained communication
1
3
4
1
2
3
4
5
(clique of size ω_2)


constrained
communication
1
2
3
4



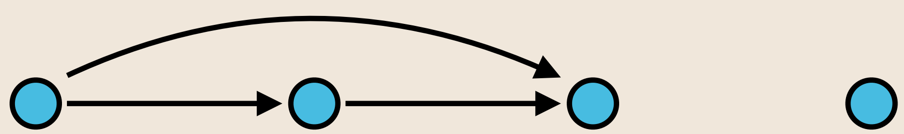
Theorem [Grimsman et al., 2018]

Consider any submodular multiagent optimization problem with strategic information exchange / processing over a series of *m* disconnected cliques. The ***optimal*** information exchange / processing satisfies

$$\frac{W(x^{\text{s-greedy}})}{W(x^{\text{optimal}})} \geq \frac{1}{2 + \sum_{i=1}^{m-1} \prod_{j=1}^{i} (1 - 1/\omega_j)} > \frac{1}{m+1}$$

Furthermore, the bound is essentially tight.

Grimsman et al., "Strategic Information Sharing in Greedy Submodular Maximization," CDC, 2018.



$$\frac{W(x^{\text{greedy}}; G)}{W(x^{\text{optimal}})} \geq \frac{1}{3}$$

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Strategic Information Exchange

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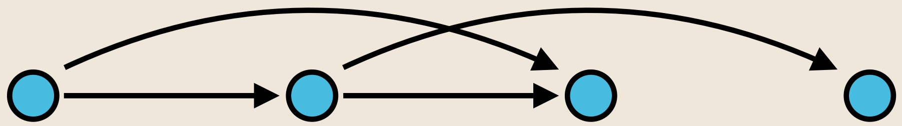

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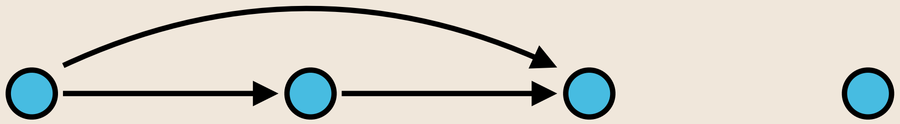
Strategic Information Exchange



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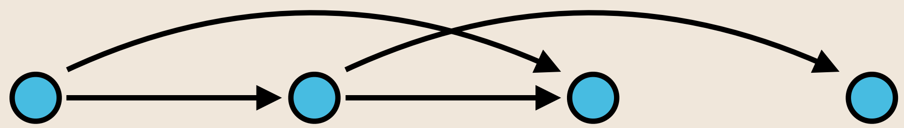

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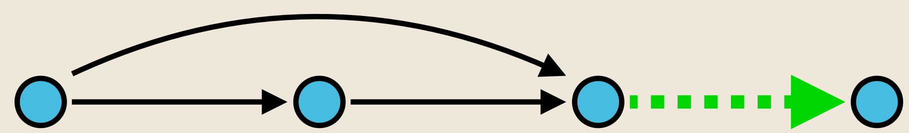

$$\frac{W(x^{\text{greedy}};G)}{W(x^{\text{optimal}})} \geq \frac{1}{3-1/3}$$

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